今後の研究計画（英訳文）

田中利史

In a study of low-dimensional topology, it is very important to characterize geometric properties of manifolds by using algebraic methods. In my future research, I set two goals as follows.

Goal 1: Studing geometric structures of 4-manifolds via Khovanov theory

In a paper published in 1984, V. F. R. Jones gave an important polynomial invariant of a knot, now called the Jones polynomial of a knot using the von Neumann algebra. On the other hand, L. H. Kauffman constructed an invariant which is equivalent to the Jones polynomial by studing a diagram of a knot or a link combinatorially. Then in a paper of M. Khovanov published in 2000, the construction of Kauffman has been generalized and a cohomology such that the Euler characteristic is the Jones polynomial was constructed from a diagram of a knot or a link via (1+1)-TQFT. E. S. Lee modified Khovanov's TQFT and gave a useful cohomology. Moreover, J. Rasmussen succeeded in constructing an effective cobordism invariant of a knot from Lee's cohomology. In fact, Rasmussen has computed the unknotting number of a torus knot by using the invariant, that is he proved Milnor conjecture combinatorially. One of my goals is to study geometric structure of 4-manifolds by using Rasmussen's invariant. We consider the following problems that are known as two of Kirby's problems.
Problem 1. Do all closed, smooth 4-manifolds have more than one smooth structure?
Problem 2. Does every non-compact, smooth 4-manifold have an uncountable number of smoothings?
In fact, we can show the existence of an exotic structure of 4-dimensional space by using Rasmussen's invariant. This fact shows that the existence of an exotic structure of a 4-dimensional space is shown by using topological methods and combinatorial methods and it is surprising because the existence of such a structure was only shown via gauge theory before. I have shown the existence of an exotic structure of a certain Casson handle using Rasmussen's invariant. I also have shown that every non-compact, connected smooth 4-submanifold of the 4-dimensional space has an exotic smooth structure. Therefore we admit the usefulness of this research. On the other hand, we have obtained similar result for any non-compact, connected smooth 4-submanifold of a connected sum of any number of CP^2. It is our assignment to consider if such 4-submanifolds admit infinitely many exotic smooth structures. We expect to be able to show that many Casson handles and some other 4-manifolds admit exotic smooth structures for giving a partial answer to Problem 1 by advancing this method. We also want to advance our reseach on Problem 2 to investigate how to apply our method to compact 4-manifolds. Moreover we want to find out what causes the richness of smooth structures peculiar to 4-manifolds topologically.

Goal 2: Finding geometric structures of 3-manifolds reflected in quantum invariants

The research related with the volume conjecture which has suggested the relation of the simplicial volume (Gromov invariant) of the complement of a knot and the colored Jones polynomial shows that a quntum invariant is deeply related to a geometric structure of a manifold. For example, the colored Jones polynomial of a figure 8 knot gives the hyperbolic volume of its complement as a certain limit. I want to investigate a relation between the Jones polynomial of a knot and a geometric structure of a 3-manifold by considering this conjecture. I have already obtained a simple formula for the colored Jones polynomial of a doubled knot. Therefore I want to study doubled knots concerning the volume conjecture.