Results

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(1) The clasp number of a knot

The *clasp number* is a knot invariant defined in 1970's, and its research may have not been developed. Professor Kadokami and I attempt to determine the clasp numbers of prime knots with up to 10 crossings. To determine the clasp number, we usually use a lower bound by the genus or the unknotting number of a knot. We focus on the Conway polynomial of a knot to deal with the case that we can not determine the clasp numbers in such a way. We prove that there exist infinitely many prime knots which can not be determined the clasp numbers in such a way by investigating algebraic properties of the Conway polynomial of a knot whose clasp number is less than or equal to two.

(2) Diagrams of embedded surface-knots and Roseman moves

An *embedded surface-knot* is a closed surface embedded in $\mathbb{R}^4$. A *diagram* of an embedded surface-knot is its image via a generic projection from $\mathbb{R}^4$ to $\mathbb{R}^3$, equipped with over/under information. Roseman introduced seven types of local moves, called *Roseman moves*, for diagrams, which generate the equivalence relation for embedded surface-knots. By investigating geometrical information of surface-knot diagrams, I solve an independence problem of seven types of Roseman moves as a local move. Moreover, jointly working with Professors Tanaka and Oshiro, we can have a result about an independence problem of the Roseman move called the *tetrahedral move* as a global move.

(3) An immersed surface-knot of ribbon-clasp type

The boundary of a handlebody immersed in $\mathbb{R}^4$ with only ribbon intersections becomes an embedded surface-knot, and such an embedded surface-knot is called *ribbon type*. Professor Kamada and I generalize a clasp intersection in 3-space into that in 4-space, and we introduce an immersed surface-knot of ribbon-clasp type. Here, an immersed surface-knot is said to be *ribbon-clasp type* if it is the boundary of a handlebody immersed in $\mathbb{R}^4$ with only ribbon intersections and clasp intersections. Moreover, we prove that analogies of geometrical properties for embedded surface-knots of ribbon type hold for immersed surface-knot of ribbon-clasp type.

(4) Quandle cocycle invariants of immersed surface-knots

For a diagram of an embedded surface-knot, one can calculate a state sum associated with a 3-cocycle of a usual quandle homology. It is known that such a state sum is an invariant of an embedded surface-knot, which is called the *quandle cocycle invariant*. To introduce the quandle cocycle invariant for an immersed surface-knot, I construct a new variation of a quandle homology. For a diagram of an immersed surface-knot, a state sum associated with a quandle 3-cocycle in a usual sense does not become an invariant of an immersed surface-knot. However, a state sum associated with a quandle 3-cocycle in a new sense becomes an invariant of an immersed surface-knot.

(5) The triple point number of an immersed surface-knot

An *immersed surface-knot* is a closed surface immersed in $\mathbb{R}^4$ generically. The *triple point number* of an immersed surface-knot F is defined by the minimum number of triple points required for a diagram of F. Professor Satoh showed that there does not exist an embedded sphere-knot whose triple point number is one, two or three. I prove that for an immersed sphere-knot with one self-intersection point, a similar result holds.