Research Results

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Spatial graph theory has been developed from Knot theory. Properties of abstract graphs determine the properties of possibility of spatial embeddings of the graphs. While knots are one dimentional manifolds, spatial graphs are one dimentional complexes, so the decision of the canonical embeddings of spatial graphs is difficult and unsolved problem except for planar graphs and so on. Kazuaki Kobayashi defined a book presentation with respect to a Hamilton path of graphs, in order to decide the canonical graphs of complete graphs which have high symmetry. In [1], I investigated two properties, H-path book presentability defined by Kobayashi in 1993, and strongly discatenability introduced by Bohme(1990), which are measures of simplicity of graphs. I showed that the two properties are not equivalent in general, but coincide for complete n-partite graphs. I also gave a necessary and sufficient condition for complete n-partite graphs to have the property.

In 1985, A.Casson introduced an integer invariant for oriented integral homology three-spheres, by using representations from their fundamental groups into $SU(2)$. It is also defined as a function of the Alexander polynomial of a framed link presenting the 3-manifold. While the topological properties of quantum invariants of 3-manifolds are known only for the special cases, an unique exception is the Casson invariant. In 1998, C.Lescop proved that any integral homology three sphere with the Casson invariant zero can be obtained from S^3 by (± 1)-surgery on a boundary link each component of which has a trivial Alexander polynomial. A 3-manifold is expressed as a surgery presentation by a framed link in S^3, and descriptions of the homeomorphic 3-manifolds are related to each other by the Kirby moves for the framed links. In [3]: From a geomoetrical view point of surgery description of homology spheres, I generalized the result of Lescop, and proved that for any integral homology 3-sphere H, there exists an integer k such that H can be obtained from S^3 by (± 1)-surgery on a boundary link each component of which has the Alexander polynomial $1 + k(t^{1/2} - t^{-1/2})^2$.

Many researchers have attempted to discover new knot invariants, however, an efficient complete link invariant has not been discovered, so far. Investigation of the precision of known invariants is required. As one way of investigation, it is effective to find links which share the same invariants. Mutation of 2-tangle is known as a deformation which preserves the skein polynomial invariants. As a generalization of mutation, the concept of 'rotation' of order n was defined as the local deformation on an n-tangle with n-rotational symmetry. Around 1989, Anstee, Rolfsen, Przytycki and Jin showed that some skein polynomials distinguish a pair of rotant links. Jin and Rolfsen conjectured, in 1989, that the rotation preserves the Alexander polynomial. In 2001, Traczyk showed that the Alexander polynomials of a pair of any orientation-preserving n-rotants coincide. In [5], we showed that there is a pair of orientation reversing rotants which do not share the same Alexander polynomial. Among skein polynomial invariants, only the Alexander polynomial is also defined independently with Seifert surface. We also showed that rotation preserves the Murasugi signature, and the Tristram-Levine signature in orientation preserving cases.

We investigated tangles in another way. In particular, we constructed a finite cyclic branched covering space associated with tangles, as an application of the Akbulut-Kirby method of construction of branched covering space branched along a knot. We introduced a disk-band representation of Seifert surface of a tangle, in order to construct branched covering space associated with tangles. Further, we gave an algorithm to give practical presentations, surgery description and Heegaad decomposition, of branched covers of tangles in [4].

Seifert surfaces played important roles in investigating tangles as above. It is also important to examine Seifert surfaces from a geometrical point of view. I exhibited the minimal genus Seifert surfaces of prime links up to nine crossings. In [6], we classified the incompressible Seifert surfaces for Pretzel links of type (p, q, r) by using the method of Kobayashi and Kakimizu.

In [7], we classified rational links, 3-braid links, and prime links up to 5-move equivalence.