Abstract

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Let $f\colon S \to \Delta$ be a proper flat morphism from a two dimensional complex manifold S to a small disk $\Delta := \{t \in \mathbf{C} | t < \varepsilon\}$. Assume that f is smooth over $\Delta \setminus \{0\}$. Let g be the genus of general fibers $f^{-1}(t)$ $(t \in \Delta \setminus \{0\})$. We call a triple (f, S, Δ) *a degeneration* of curves of genus g. We call $f^{-1}(0)$ the *special fiber*. If all general fibers are hyperelliptic, we call such a degeneration a *hyperelliptic degeneration*. For each topological type of degenerations, we can determine the conjugacy class of the mapping class group of genus g. We call it the *topological monodromy* (monodromy, for short).

In [5], in the case where $g = 3$, we classified the configurations of singular fibers, topological monodromies and topological types of their stable models using the theory of Matsumoto-Montesinos. We also proposed an algorithm to classify the topological monodromies of degenerations of curves in any genus. Note that our result was due to the topological method which was not used in the case $g = 1, 2$ studied by Kodaira and Namikawa-Ueno.

To observe the influence of the complex structures of general fibers for the monodromies, we classified completely the topological monodromies of hyperelliptic degenerations of genus three ([4]). Moreover, for each monodromy $[\phi]$, I gave a defining equation of a hyperelliptic degeneration whose monodromy is $[\phi]$.

In [3], I classified periodic monodromies(maps) which commute with the hyperelliptic involution using the method of algebraic geometry.

Using the similar techniques in [3], we proved that each semistable monodromy which commutes with a hyperelliptic involution is realized as the monodromy of a hyperelliptic degeneration ([2]). I think that this result would not be proved by techniques of topology. Thus, this result is one of the examples that the methods of algebraic geometry contributed to a study of topology.

As is well-known, any element of mapping class group can be written as a composition of Dehn twists. The presentations of elements of mapping class group obtained from splitting families are called *geometric presentations*. Cadavid conjectured that any geometric presentation is the shortest presentation by Dehn twists. In [1], using a splitting family of $g = 2$, I constructed a counter example of Cadavid conjecture.

In [7], using Matsumoto's method(splitting families), I proposed an algorithm to obtain the presentations of periodic maps which commute with the hyperelliptic involution.