

The plan for the study

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We study general submanifolds in a complex space form with respect to the condition on some kinds of curvatures or the second fundamental form without the assumption that the submanifolds are complex, totally real or CR .

For the study of general real submanifolds in a complex space form, the conditions like 'totally umbilical' and 'parallel second fundamental form' do not work well. So we need other essential geometrical conditions instead.

Especially, we study the following problems:

(I) Pinching problems for compact minimal submanifolds in a complex projective space.

We improve our pinching results for compact minimal submanifolds in a complex projective space with respect to the scalar curvature and the sectional curvature proved in [4] and [6].

In order to give another Simons type integral formula, for example, we compute the Laplacian for the square of the length of the second fundamental form in the direction of the mean curvature vector. We use such formulas to solve pinching problems.

On the other hand, in order to find new examples of submanifolds, we need new geometrical quantity. We consider the geometrical meaning of some characteristic terms appeared in our theorems and their proof.

We also study pinching problem for compact submanifolds in a complex space form of nonnegative holomorphic sectional curvature with parallel mean curvature vector field.

(II) The Ricci tensor of a real hypersurface on the holomorphic distribution.

In [3], we studied real hypersurfaces in a complex space form whose second fundamental form satisfies certain condition on the holomorphic distribution which extends the notion of 'totally η -umbilical'. We continue this approach to characterize some real hypersurfaces with respect to the conditions on the holomorphic distribution.

First, we improve the results in [7]. We study some conditions for the Ricci tensor on the holomorphic distribution to extend the known notions such as pseudo-Einstein.

(III) The second fundamental form of real submanifolds in a complex space form.

For real submanifolds in a real space form and complex submanifolds in a complex space form, there are many results about the condition that the second fundamental form is parallel. On the other hand, if the second fundamental form of a submanifold in a complex projective space is parallel, then the submanifold is complex or totally real. So we need another suitable condition for the second fundamental form of real submanifolds in a complex space form instead. Moreover, it is important to consider the case when the ambient manifold is a complex space form of negative holomorphic sectional curvature.

Combining these, we study general submanifolds in a complex space form including complex submanifolds, totally real submanifolds and CR submanifolds. Especially we aim to find new kinds of examples of submanifolds in a complex space form.