Research plan

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I plan to study the structure of self-homotopy and L-S category theory by Whitehead product and Hopf invariant.

The self-homotopy of a suspended space has a group structure by the homotopy addition induced from the co-group structure of the suspension. Moreover it has a monoid structure by the composition of maps. In general, the self-homotopy together with those operations is not a ring since it does not satisfy the right distributive law. There is a structure of the self-homotopy together with those operations, called square ring, which is introduced by H. J. Baues, M. Hartl and T. Pirashvili. We can consider a module over a square ring like a module over a ring. I think that we can examine the homotopy set more deeply via module over square ring. But its complexity, it is not researched so much. So, I would like to research the square ring and the module over it with $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ as an example. For $k \geq n$, it is well-known that $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ is a ring by the Freudenthal suspension theorem. The square ring is related with Whitehead products and Hopf invariants. For $k \geq n$, Whitehead products and Hopf invariants vanish in $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$, and $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ is a trivial square ring. In my research, there are $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ which are a ring in $k < n$. Though, in these case, there is a difference between $k < n$ and $k \geq n$ as a square ring. This is due to that either Whitehead products or Hopf invariants does not vanish. However, there are few cases which determined the square ring structure of $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$. I think that I determine the structure of $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ by advancing research of Whitehead products and Hopf invariants. Accordingly, I will be able to observe $[\Sigma^k \mathrm{P}^n, \Sigma^k \mathrm{P}^n]$ becomes a ring. Moreover, I want to give the structure of the self-homotopy which is not a square ring.

In research of L-S category theory, as the first step, I think the evaluation method of determining the L-S category of SO(10) is enhanced. This evaluation uses the structure as the attaching space of the product space. And, the importance is the attaching map described by the Whitehead product. Moreover, Hopf invariant is one of the conditions that this evaluation method can be used. Also in research of L-S category theory, by advancing research of Whitehead products and Hopf invariants, I would like to determine the L-S category of the Stiefel manifold using this extended valuation method.