1 Subject and Achievement of past Research

My research covers several areas of knot theory, with relations to combinatorics, number theory and algebra, which are outlined below (and by no means unrelated).

1.1 Vassiliev invariants

My first work was to improve the upper bound on the dimension of Vassiliev invariants of degree D. The best known previous upper bound was $(D-2)!/2$ due to Ng. It was known that Vassiliev invariants can be understood combinatorially in terms of chord diagrams modulo the $4T$ relation. I introduced a certain type of chord diagrams and showed that they generate modulo $4T$ relations the space of Vassiliev invariants. Then I estimated from above the number of such chord diagrams to $D!/1.1^D$. Later, Zagier showed that the generating series of the numbers of such chord diagrams occurs in a strange identity related to the Dedekind eta-function. He found the exact asymptotical behaviour of these numbers, improving the number 1.1 to $\pi^2/6$, thus establishing the currently best upper bound.

My later work were constructions of knots with Vassiliev invariants of bounded degree and specific properties, like given unknotting numbers, signatures and 4-genera. I showed the non-existence of Vassiliev invariants that depend on any finite number of link polynomial coefficients (except the Conway/Alexander polynomial).

1.2 Legendrian knots

Legendrian knots are called knots embedded in the standard contact space. There are inequalities relating the Thurston-Bennequin invariant and Maslov number of Legendrian knots and the degrees of the polynomial invariants of the underlying topological knots. Using these inequalities I gave estimates of the invariants of Legendrian negative knots. This result can be considered as a generalization of Kanda's determination of the maximal Thurston- Bennequin invariant of the negative trefoil.

1.3 Gauss diagram formulas

Fiedler and Polyak-Viro introduced a new approach to defining Vassiliev invariants by explicit formulas. Such formulas involve sums over specific tuples of crossings of a knot or link diagram of functions involving writhes of the crossings (similarly to linking numbers). These formulas proved useful in the study of positive knots (knots with diagrams all of whose crossings are positive). Positive knots and links have been studied, beside because of their intrinsic knot-theoretical interest, with different motivations and in a variety of contexts, including singularity theory, algebraic curves, dynamical systems, and (in some vague and yet-to-be understood way) in 4-dimensional QFTs. Using the Fiedler-Polyak-Viro formulas, I found several inequalities between Vassiliev invariants of positive knots, allowing to exclude certain knots from having this property. Later I sought generalizations of some criteria to almost positive knots.

1.4 Canonical Seifert surfaces

The set of knot diagrams whose canonical Seifert surfaces (that is, surfaces obtained by Seifert's algorithm) of given (canonical) genus admits a structure of generating series. It allows to prove, for example, that the number of alternating knots of fixed genus grows polynomially in the crossing number.

1.5 Non-trivial Jones polynomial problem

In 1985 Jones discovered the famous polynomial invariant named after him and asked if it distinguishes all non-trivial knots from the trivial one. His question remains unanswered despite the recent (negative) solution for links. I showed that semiadequate links, as defined by Lickorish-Thistlethwaite, have non-trivial Jones polynomial. Montesinos links are semiadequate, and then I showed that so are 3-braid links, so the non-triviality result applies to these classes.

1.6 Closed 3-braids

I classified among closed 3-braid links the braid positive, strongly quasipositive and fibered ones. Then I showed that 3-braid links with given (non-zero) Alexander or Jones polynomial are finitely many, and can be effectively determined. In recent joint work with M. Hirasawa and M. Ishiwata we showed that 3-braid links have a unique incompressible Seifert surface.

1.7 Knot tables

For some time I have been interested in using knot tables, compiled by Hoste, Thistlethwaite, and Weeks, to seek knots with interesting properties, and so to provide examples and counterexamples to problems that have remained inaccessible using (entirely) manual reasoning.

1.8 Other topics

I have also done some work on unknotting numbers, link polynomials, number theoretic properties of knot invariants, and enumeration problems of links.