Plan of Research

Ryosuke Yamamoto

Giroux's one-to-one correspondence between open book decompositions and contact structures of closed oriented 3-manifolds is playing an important role to study both structures. On the study of open book decompositions, the solution of Harer's conjecture is its great contribution. On the study of the contact structures, a characterization of contact structures through corresponding open book decompositions has become a very powerful tool. I plan the following:

Overtwisted contact structures and open books

Recent research on contact structure of 3-manifold is based on dividing them into two classes, tight contact structures and overtwisted ones. I showed the following as mentioned in "Summary of my research": "A given open book decomposition is corresponding to an overtwisted contact structure if and only if we may positively stabilize it to be an open book decomposition with Stallings twist." In the proof of this, I showed a technique of positive stabilizaitions detecting overtwistedness. I expect that analysing this technique leads to a criterion of overtwistedness without stabilizations.

Monodromy maps of open books corresponding tight contact structures

Monodromy map of an open book decomposition for a closed oriented 3-manifold is represented by a product of Dehn twists on the fiber surface. It is known that an open book decomposition is corresponding to a tight contact structure if its monodromy can be represented a product of only positive Dehn twists, and also known that the reverse is not true. I will investigate negative Dehn twists on a fiber surface to understand when they break the tightness of corresponding contact structures.

Murasugi decomposition of fiber surface and contact structure

It has been a long-pending problem to find a criterion for whether a fiber surface is decomposable with respect to Murasugi sum. I plan to study for characterization of the contact structure on S^3 supported by an open book decomposition with fiber surface which is not decomposable with respect to Murasugi sum.

On a complexity of open book decompositions

In a joint work with Saito, we introduced the *translation distance* of an open book, which is the minimal length of translation of vertices of arc complex on the fiber under the monodromy map. I will investigate this complexity of open books in detail. Furthermore I will study the structure of the arc complex of the fiber and the action of the monodromy on it to find a measure which represents the topology of open books more precisely.

Knots in S^3 with lens surgery

Berge gave a listing of knots in S^3 which admits a Dehn surgery yielding a lens space, called "lens surgery". It is conjectured that a knot in S^3 admits lens surgery if and only if the knot is in Berge's list. I plan the following to solve the conjecture.

Recently it was shown that any knot in S^3 which admits lens surgery is fibered one. Therefore we may consider a contact structure on S^3 supported by the open book with such a knot as the binding. Through the lens surgery we also have a contact structure on lens space. I will analyze a relation between the contact structure and the genus 1 Heegaard splitting of lens space. I expect that it would develop a new approach to study knots with lens surgery.