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[Our previous research]

[(1) On 4-cocycles of Alexander quandles on finite fields] Quandle shadow cocycle invariants are invariants of oriented surface-links. To calculate these invariants, we need concrete quandle 4-cocycles. For Alexander quandle X on the finite field, we represented a non-trivial 4-cocycle as a polynomial, and when this quandle is $H_Q^2(X;\mathbb{Z}) \cong 0$, we decided $H_Q^4(X;\mathbb{Z})$. This research has been developed by Nosaka Takefumi. Quandle homotopy invariants which are universal invariants of quandle (shadow) cocycle invariants have a value in the homotopy group $\pi_3(BX)$ of the third degree that is a quandle classifying space. In case of Alexander quandles on the finite field (generally regular quandles can be used.), by the universal coefficient theorem and the hurewicz fundamental homomorphism theorem, we gained $H_Q^2(X;\mathbb{Z}) \cong 0 \Rightarrow H_Q^4(X;\mathbb{Z}) \cong \pi_3(BX)$, and thus the generating elements of quandle homotopy invariants of surface links were decided.

[(2) Willerton conjecture] Generally, it is known that when for the normalized prime vassiliev invariant v_d of the degree d, the knot K has a diagram of n crossings, the value of $v_d(K)$ is shown by the order of n^d. Therefore, the set $\left\{ \left(\frac{v_2(K)}{n^2}, \frac{v_3(K)}{n^3} \right) \in \mathbb{R}^2 \middle| K \text{ has a knot diagram with } n \text{ crossings} \right\}$ is bounded. For some knots, points are plotted, and then a fish-link graph appears. We found what shape this graph could be for torus knots. Moreover, for these knots, we completely solved the problems Willerton conjecture posed.

[(3) Relation between quandle (shadow) cocycle invariants and finite type invariants] The relation between quandle (shadow) cocycle invariants and quantum invariants is not well known except the set the relation of the set-theoretic Yang-Baxter equation. We showed that a kind of special finite type invariants could be gained from quandle (shadow) cocycle invariants by giving filtration to knot determinant. Moreover, by applying this to three-manifold, we redefined finite type invariants different from Ohtsuki invariants for general three-manifold. It was shown that these invariants are calculable for a concrete lens space and for Brieskorn manifold, and are not empty sets. These invariants are stronger at least than Dijkgraaf-Witten invariants.

[(4) Perturbative $\mathfrak{sl}_2$ invariants of handlebody-knots] We gained the perturbative $\mathfrak{sl}_2$ invariants from the quantum $U_q(\mathfrak{sl}_2)$ invariants of handlebody-knots which had already been defined. The quantum $U_q(\mathfrak{sl}_2)$ invariants are gained by replacing edges and trivalent vertexes with the copies of the links, taking the linear sums of Jones polynomials of those links and substituting the complex root of unity 1 (which is similar to the quantum $SU(2)$ invariants of three-manifold). At first, we considered that the invariants as strong as quantum $U_q(\mathfrak{sl}_2)$ invariants would be found. However, by normalizing them and limiting the sum to gain perturbative $\mathfrak{sl}_2$ invariants, the invariants stronger than quantum $U_q(\mathfrak{sl}_2)$ invariants were found. In fact, the two handlebody knots to which a complement is homeomorphic cannot be distinguished by using usual quantum $U_q(\mathfrak{sl}_2)$ invariants. However, those handlebody-knots can be distinguished by using newly defined perturbative $\mathfrak{sl}_2$ invariants.