Summary of Research

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1. Sigma function of telescopic curves

The elliptic sigma function possesses the following properties. 1. Any elliptic function can be written as a rational function of $\wp$ and $\wp'$, where $\wp$ is the second logarithmic derivative of the sigma function. 2. The functions $\wp$ and $\wp'$ realize the elliptic curves in the standard Weierstrass model. Klein constructed sigma functions of many variables which possess analogues of the properties 1 and 2 for hyperelliptic curves. For hyperelliptic curves of genera 2 and 3, Baker obtained explicit expressions for higher logarithmic derivatives of sigma functions of many variables in the form of polynomials in the second and third logarithmic derivatives of these functions. These differential polynomials give fundamental equations of mathematical physics, including the KdV and KP equations. Buchstaber, Enolski, Leykin extended the sigma function to more general curves called (n, s) curves [1]. In [5] it is shown that the coefficients of series expansion of the sigma function of (n, s) curves around the origin are polynomials of the coefficients of the defining equation with rational numbers. This fact, which plays an important role in applications, distinguishes the sigma function from the Riemann theta function. We extended the sigma function to telescopic curves, which are introduced in [4] and contain the (n, s) curves, and showed that the same properties of series expansion (publication list 1-3,4). We extended the addition formulae of sigma function of (n, s) curves [6] to telescopic curves, which is joint work with A. Nakayashiki (publication list 1-4).

2. A generalization of Jacobi inversion formulae to telescopic curves

Via the Abel-Jacobi map, the field of meromorphic functions on the Jacobian of an algebraic curve (Abelian functions) is isomorphic to the field of rational functions of the algebraic curve. The rational functions are computable and we can derive relations among them. By transferring the relations to the Abelian functions, we can obtain the relations among them, which are the differential equations integrable by the Abelian functions. It is important to clarify the correspondence of the isomorphism. For hyperelliptic curves, the rational functions expressed by the fundamental symmetric polynomials of the coordinates of points correspond to the hyperelliptic functions, which are defined by the logarithmic derivatives of the sigma function (Jacobi inversion formulae). In [3], the formulae are extended to the more general curves defined by $y^r = f(x)$. We extended the formulae to telescopic curves (publication list 1-2). More specifically, for telescopic curves, we gave the rational functions of the curve corresponding to the Abelian functions defined by the logarithmic derivatives of the sigma function explicitly.

3. Polynomial dynamical systems integrable by the sigma function

It is well known that the field of meromorphic functions on the Jacobian of a hyperelliptic curve is generated by the hyperelliptic functions and the relations among the generators are derived. The zero set of the sigma function in the Jacobian is called sigma divisor. In this research we constructed a generator of the field of meromorphic functions on the sigma divisor of a hyperelliptic curve of genus 3 in terms of the sigma function and derived all the relations among the generators. As an application, we constructed a solution of the dynamical systems introduced in [2] in terms of the meromorphic functions on the sigma divisor, which is joint work with V. M. Buchstaber (publication list 1-1).

References

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