Research plan

Knotted surfaces as algebraic varieties

The work in [2] provides an algorithm that constructs a polynomial $f:\mathbb{R}^4 \to \mathbb{R}^2$ whose vanishing set intersects the unit three-sphere in any given link $L = f^{-1}(0) \cap S^3$. I plan to generalize this algorithm to higher dimensional maps $f:\mathbb{R}^5 \to \mathbb{R}^2$, providing a method to construct polynomial maps that vanish on any given knotted surface $K = f^{-1}(0) \cap S^4$.

While the original construction in [2] has proven useful for the creation of knotted initial configurations of physical systems [1], this higher dimensional analogue could provide more control over the time-evolution of these systems, using the fourth dimension of the ambient space S^4 as a time coordinate.

Relations between knotted surfaces and the corresponding polynomials

As in the lower dimensional case in [2] we should expect relations between properties of the knotted surfaces and the corresponding polynomials, such as a bound on the polynomial degree in terms of numbers that are usually associated with the complexity of the knotted surface. In general, this project amounts to investigating which surfaces can be constructed if we impose extra conditions on the polynomials f, such as holomorphicity, the existence of an isolated singularity or that $\arg f$ is a fibration of $S^4 \backslash f^{-1}(0)$ over S^1.

Crossing numbers of composite knots

I will continue to investigate relations between the crossing numbers of composite knots and certain spatial graphs with a view toward the conjecture about the additivity of the crossing number [4]. Especially knots that are of the form $K\#K\#\ldots\#K$ for some knot K form a promising family. I would also like to improve lower bounds of crossing numbers (such as Lackenby's [5]) that hold for all knots.

References

[1] B Bode, MR Dennis, D Foster, RP King. Knotted fields and explicit fibrations for lemniscate knots. *Proc R Soc A*. **475** (2017), 20160829.

[2] B Bode, MR Dennis. Constructing a polynomial whose nodal set is any prescribed knot or link. *arXiv:1612.06328*.

[3] B Bode. Constructing links of isolated singularities of real polynomials $\mathbb{R}^4 \to \mathbb{R}^2$. *arXiv:1705.09255*.

[4] B Bode. Crossing numbers of composite knots and spatial graphs. *Topology and its Applications*. **243** (2018), 33–51.

[5] M Lackenby. The crossing number of composite knots. *Journal of Topology*. **2** (2009), 747–768.