

# Summary of Research

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My main research interest is low dimensional topology, especially, study of *smooth 4-manifolds* via *mapping class groups of surfaces*. More specifically, finding new *Lefschetz pencils* with various interesting features has been my greatest contributions to the research area and I take pride in having distinguished skills to construct Lefschetz pencils from the combinatorial aspect in terms of mapping class groups. Lefschetz pencils can be regarded a topological description of symplectic manifolds, hence my interest naturally extends to *4-dimensional symplectic topology*, and also to *3-dimensional contact topology* (as the boundaries of symplectic 4-manifolds).

*Lefschetz pencils* were originally introduced to study topological aspects of non-singular projective varieties, so *holomorphic* Lefschetz pencils had been of classical interest. However, it revealed in the late 1990s that there is an excellent correspondence between Lefschetz pencils and *symplectic manifolds*, and since then this fact has become the strongest motivation to study Lefschetz pencil structures. Namely, the following fundamental theorem holds: a closed manifold admits a symplectic structure if and only if it admits a Lefschetz pencil structure. From this fact, we can study symplectic manifolds via Lefschetz pencils.

I have mainly worked to construct explicit Lefschetz pencils on basic symplectic 4-manifolds or construct Lefschetz pencils (or Lefschetz fibrations) with interesting features. In the paper [1], I studied the minimal numbers of singular fibers in Lefschetz fibrations over the tori, and gave pretty good upper bounds for them. The paper [2] investigates holomorphic Lefschetz pencil structures on the four-torus with great details. The preprint [3] presented systematic constructions of various examples of non-holomorphic Lefschetz pencils. In [4], I elaborately studied the pencil structures of one of the most basic Lefschetz fibrations called the Matsumoto-Cadavid-Korkmaz Lefschetz fibration and gave various Lefschetz pencils. In [5], I discovered much simpler expressions for the well-known genus-1 Lefschetz pencils.

### REFERENCES

- [1] N. Hamada, *Upper bounds for the minimal number of singular fibers in a Lefschetz fibration over the torus*, Michigan Math. J. **63** (2014), 275–291.
- [2] N. Hamada and K. Hayano, *Topology of holomorphic Lefschetz pencils on the four-torus*, to appear in Algebr. Geom. Topol.; also available at <https://arxiv.org/abs/1603.08284>.
- [3] N. Hamada, R. Kobayashi and N. Monden, *Non-holomorphic Lefschetz fibrations with  $(-1)$ -sections*, preprint; <https://arxiv.org/abs/1609.02420>
- [4] N. Hamada, *Sections of the Matsumoto-Cadavid-Korkmaz Lefschetz fibration*, preprint; <https://arxiv.org/abs/1610.08458>
- [5] N. Hamada, *Simple expressions for the holed torus relations*, preprint(2017); <https://arxiv.org/abs/1701.02171>.