Feature work

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We will study about region crossing change, region freeze crossing change and Reidemeister move on spherical curve.

For knots and links, there are many researches with invariants. In particular, Vassiliev[Va] introduced an invariant called Vassiliev invariant in 1990. It gives a certain classification of knot invariants. In fact, there is a conjecture called Vassiliev Conjecture;

For a pair of knots K_1 and K_2, if K_1 does not equal to K_2, then Vassiliev inv. distinguishes K_1 and K_2 .

K_1 equals to K_2 if and only if a diagram of K_1 obtained from a diagram of K_2 by Reidemeister move (see Figure 1). Then, on this move and Vassiliev invariant, we have following fact;

If a value of Vassiliev invariant for a diagram D_1 equals to a value of Vassiliev invariant for a diagram D_2 applied $\Omega 1$ and $\Omega 3$, then the values of Vassiliev invariant are the same.

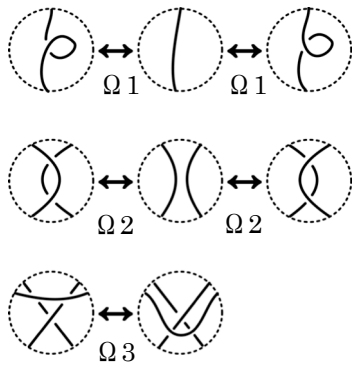

Figure 1

Since the previous fact, we expected that if for a pair of link diagrams D_1 and D_2, D_2 is obtained from D_1 by $\Omega 1$ and $\Omega 3$, then D_2 is obtained from D_1 by $\Omega 2$. However, Hagge-Yazinski[HY] gave an example from which the simple closed curve is NOT obtained by $\Omega 1$ and $\Omega 3$.

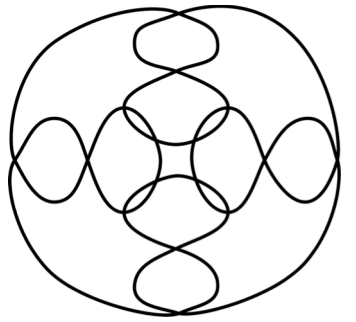

Figure 2.

An example of Hagge-Yazinski

Viro [V] proposed Reidemeister moves of spherical curves called RI, weak RII, strong RII, weak RIII and strong RIII. Then we consider following problem;

We consider equivalence relations induced by a subset of {RI, weak RII, strong RII, weak RIII and strong RIII}. Then characterize equivalence classes by equivalence relations

For a projection G arising from a link diagram D, we define a 1-dimensional simplicial complex $C(G)$ as follows.

> A 0-complex of $C(G)$ corresponds to an element of the power set of vertices of G. There is an edge e such that e connects two vertices v,v' of $C(G)$ if and only if there is a region R such that "the symmetric difference of the set of vertices of G corresponding to v, and the set of vertices of G corresponding to v' " is changed by the region crossing change at R.

We develop the theory from the viewpoint "the shape of the simplicial complex". In particular, we give a global understanding of studies about an efficiency of the switching system introduced by Akio Kawauchi-Kengo Kishimoto-Ayaka Shimizu, and Ayumi Inoue-Ryo Shimizu's results [IS]. Particularly, Kawauchi-Kishimoto-Shimizu introduced a game based on region crossing change. We would link to introduce the difficulty and the efficiency of the switching system. Here is an example of the usage the complex: by a result of A. Shimizu, we see that $C(G)$ is connected (in example of the figure, we remark $C(G)$ contains 2 components). On the other hand, we define a simplicial complex $C'(G)$ by region freeze crossing change introduced by Inoue-Shimizu. Then results of Inoue-Shimizu is regarded as "there exists a projection represented of knot such that $C'(G)$ is not connected". For a mutant of region crossing change, we obtain a simplicial complex from the operation. Then it shows some properties of the operation. Then we develop this philosophy.

Reference

[Va] V. A. Vassiliev, Cohomology of knot space, in Theory of Singularities and its Applications (ed. V. I. Arnold), Adv. Soviet Math. 1, Amer. Math. Soc., 1990.

[Vi] O. Viro, Generic immersions of the circle to surfaces and the complex topology of real alge- braic curves. Topology of real algebraic varieties and related topics, 231–252, Amer. Math. Soc. Transl. Ser. 2, 173, Adv. Math. Sci., 29, Amer. Math. Soc., Providence, RI, 1996.