Research Results --- Yasuo Matsushita
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One of the major subjects of Professor Matsushita is the differential geometry and topology of indefinite metric spaces. The main results are as follows:

1. The existence condition for a compact oriented 4-manifold to admit a neutral metric of signature (＋＋— —) are settled. In relativity theory the existence condition of a Lorentz metric on a manifold is well-known. Therefore, the problem of the existence condition for the indefinite metric of neutral signature on 4-manofolds is interesting because such a metric is the lowest dimensional example of indefinite metrics not of Lorentz type. Its existence condition is known as same as the existence of a field of 2-planes. Applying Theorem of Hirzebruch and Hopf (1951) and Donaldson's works, celebrated as the Fields Prize in 1986, the existence condition of a neutral metric on a 4-manifld is expressed in terms of the Euler characteristic and the Hirzebruch index of the manifold, which are both topological invariants. These results are cited in Donaldson's Book "The Geometry of Four-Manifolds," published by Oxford University Press, in 1990.
2. One of the other significant results is to show a counter example constructed on a Walker 8-manifold to the Goldberg conjecture, posed in 1969, which states that an almost complex structure of a compact Einstein almost Kaehler manifold must be integrable, in other words, a compact Einstein almost Kaehler manifold must be Kaehler.
3. It is shown that the Euler characteristic and the Hirzebruch index must of a compact neutral Einstein 4-manifold must obey an inequality, which is similar to the Hitchin-Thorpe inequality for Riemannian-Einstein 4-manifolds, only except the sign.
4. Moreover, it is proved that such an existence condition of a neutral metric on a 4-manifold is equivalent to the existence condition of a pair of an almost complex structure and an opposite almost complex structure on the 4-manifold.
5. On the basis of the Enriques-Kodaira classification of compact 2-dimensional manifolds, usually called *surfaces*, it is shown that a surface admits an opposite almost complex structure if and only if the second Chern class of the surface is even.
6. Petean reported a new example of a neutral Einstein Kaehler 4-manifold. A general method of constructing new such examples of neutral Einstein Kaehler 4-manifolds are proposed in terms of arbitrary 2-dimensional harmonic functions.
7. Generalizing the notion of null vectors, we proposed isotropic tensors, with property of the zero squared norm of tensors. As one of the significant examples, a new example of isotropic Kaehler structure is constructed on an Engle 4-manifold.
8. Recently, some substantial results are obtained on the spinor approach to neutral geometry of 4-manifolds.
9. In 2007, a counterexample to the Goldberg conjecture is discovered on an 8-dimensional Walker manifold of neutral signature (+4, -4).
10. In 2015, such a counterexample is exhibited on a 6-dimensional Walker manifold of signature (+4, -2).
11. Then, in 2016, a counterexample to the Goldberg conjecture is discovered on an 8-dimensional Walker manifold of neutral signature (+4, -4), which are associated with

the opposite almost complex structures.

The other activities:

- •Mathematical Reviews Reviewer, from 1981.
- •The Editor-in-Chief of JP Journal of Geometry and Topology, from 2006.
- •An Interview, with a photo, by Institute Of Physics of Great Britain was opened during several years in the IOP Home Page.
- •An External Examiner for PhD evaluation in King Saud University in Saudi Arabia, in 2009.
- •A Chairperson of International Meeting in India, in 2010.
- •Invited Plenary Lecture at Santiago de Compostela University, in 2010.
- •Invited Lecture at Vienna University of Technology, in 2010.
- •A Keynote Speaker at Geometry Symposium XI in Turkey, in 2013.
- •Invited Plenary Lecture at Daejeon, Korea, in 2015.
- •Invited Lecture at Kyunpook National University, Daegu, Korea, in 2016.
- •Invited Plenary Lecture at Bulgarian Academy of Science, Sofia, Bulgaria, in 2016.
- •Invited Lectures at Granada University and Santiago de Compostela University, Spain, in 2017.
- •Invited Intensive Lectures on Indefinite Manifolds are given at Kyunpook National University, Daegu, Korea, February 2018.