Research plan

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Hilbert Geometry. Oguni and I proved that the asymptotic dimension of two dimensional Hilbert geometry is two. It follows that the coarse Baum–Connes conjecture holds for any two dimensional Hilbert geometry, which is stronger version of the coarse Novikov conjecture. We expect that this conjecture holds for arbitrary dimensional Hilbert geometries. The difficulty is to get the bound of the asymptotic dimension from below. This happens when a Hilbert geometry has both Gromov hyperbolic and Euclid like geometric structure simultaneously. To get over this, we detect which part of the space has Euclid like geometric structure by Karlsson and Noskov's characterization of Gromov hyperbolicity of the Hilbert geometries.

Teichmüller space. I am considering about coarse geometric aspects of the Teichmüller space. It is important and natural to prove the existence of coronae, which is good boundaries in the sense of coarse geometry. To do this, I show that the space is coarsely isometric to a Hilbert geometry at first. And I apply my study of the coarse geometry of the Hilbert geometry with Oguni. As a consequence of this, I will show that the Thurston boundary is a corona. This approach is related to the problem given by Papadopoulos, that is, realize the Teichmüller space as a bounded convex domain and investigate the Hilbert geometry on that domain.

Outer automorphisms of free groups. I am studying the dynamics of a fully irreducible element by looking at the behavior of its commensurability class. Indeed, the boundary point corresponding to a fully irreducible element is a commensurability invariant. I need to treat deformation spaces corresponding to each fully irreducible elements in the commensurability class uniformly. Thus it will be useful that an analogue of the Teichmüller space of a solenoid defined by Sullivan. The commensurator group of the base surface group acts naturally on this space. Originally, Sullivan is motivated to study the solenoids because of the connection with the Ehrenpries conjecture which is stated for any two closed non-conformal Riemann surfaces of the same genus greater than 1 and for any $\epsilon > 0$ there exist two finite sheeted, unbranched, conformal covers that are $(1+\epsilon)$–quasiconformal. This conjecture can be stated in the terminologies of solenoids as follows. The action of the commensurator group of the base surface group on the Teichmüller space of the compact solenoid has dense orbits. I expect that the same approach can be applied to the Ehrenpreis conjecture for graphs. The Ehrenpreis conjecture for graphs is stated as follows. For any two graphs there exists two finite sheeted covers which are arbitrary close to each other in the sense of (symmetrized) Thurston metric.