Results of my research

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A handlebody-knot is a handlebody embedded in the 3-sphere, denoted by H. We defined the Alexander polynomial of a pair of a genus g handlebody-knot H and its meridian system $M = \{m_1, m_2, \ldots, m_g\}$. We introduced a Seifert complex and a C-complex of a handlebody-knot and its meridian system M. Let $\Gamma := l_1 \cup l_2 \cup \cdots \cup l_g$ be a spatial g-bouquet which represents H, and $v := l_1 \cap l_2 \cap \cdots \cap l_g$ the vertex of Γ. A meridian system M of Γ is *standard meridian system* of Γ if m_i is the meridian of l_i $(1 \leq i \leq g)$. A spatial g-bouquet Γ is *standard spatial g-bouquet* for (H, M) if M is the standard meridian system of Γ. Then the following lemma holds.

<u>Lemma 1</u>
For any (H, M), there exists uniquely the standard spatial g-bouquet Γ.

A union of Seifert surface $S^g = S_1 \cup S_2 \cup \cdots \cup S_g$ is a *g-leafed Seifert complex* of (H, M) if S_i is a Seifert surface of l_i $(1 \leq i \leq g)$ and S_i and S_j intersect transversely except for v $(i \neq j)$. A union of Seifert surface C^g is a *g-leafed C-complex* of (H, M) which has only clasp singularities and no triple points except for v. We introduced a method to calculate the Alexander invariant for a handlebody-knot by a C-complex.

We show that an equivalent class of a C-complex characterizes (H, M). Let C^g and $C^{g\prime}$ be g-leafed C-complexes. C^g and $C^{g\prime}$ are equivalence if C^g can be transformed to $C^{g\prime}$ by a sequence of $(I0)$, $(I1)$, $(I2)$, $(I3)$ and $(I4)$-moves and $\overset{h}{\sim}$, denoted by $C^g \sim_* C^{g\prime}$.

<u>Theorem 2</u> [O.]
$\{(H, M)\} \overset{1:1}{\longleftrightarrow} \{C^g\}/\sim_*$

$(I0)$ ambient isotopy of Seifert complex