

Plan of Research

Takeshi Oota

The work of Alday-Gaiotto-Tachikawa in 2009 leads to a renewed attention to the connection between two-dimensional conformal field theories and four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories. The two-dimensional correlation function (conformal block) is identified with the instanton partition function of the gauge theory.

This “2d/4d connection” has been generalized to various directions. One of these generalizations is a correspondence between the “ q -deformed” two-dimensional conformal field theories and the “ q -lifted” five-dimensional supersymmetric gauge theories. The q -deformed 2d theories have q -Virasoro or q -W algebras, while for the q -lifted 5d theories, the 5-th direction is compactified on a circle S^1 whose radius is proportional to $\log q$.

In the 2d theory, the q -Virasoro algebra plays a central role. Its representation theory is still not well understood although 20 years have passed since its introduction. In particular, for a q -deformation of primary fields in the conformal field theory, the first principle definition of q -vertex operators is not known.

In our work, we showed that by taking the root of unity limit of q , the parafermion algebra can be obtained from the q -Virasoro or q -W algebras. We will also study this limit in more detail, and try to elucidate various properties of the 2d/4d connection. One of future theme of our study is the extension to the gauge theories on general ALE spaces or to quiver gauge theories. It is proposed an construction of associated “Yangian” for the quiver gauge theories. In the Schur-Weyl duality, the Yangian is connected with a Hecke algebra. Hence, an extension to these direction will deepen our understanding of the gauge/CFT/matrix model correspondence.

The “2d/5d connection” leads to various “2d/4d connections”. Similarly, this “2d/5d connection” may be obtained from more fundamental connection. We will try to understand these connections from wider perspective.

Furthermore, it is known that the q -Virasoro and q -W algebras have close connection with a Hopf algebra, called the Ding-Iohara-Miki (DIM) algebra with a particular structure function. This structure function admits a generalization by introducing one more parameter p to obtain the so-called elliptic Ding-Iohara-Miki algebra. It is natural to expect that the “elliptic Virasoro algebra” and “elliptic W algebras” related with this elliptic DIM algebra play the crucial role in “2d/6d connection”: the 2d correlation function with elliptic Virasoro/W symmetry is related with the partition function of 6d $\mathcal{N} = (2, 0)$ superconformal theories. We will investigate this connection. If this “2d/6d connection” holds, we will also study roots of unity limit of parameters in these elliptic algebras.

In addition to the DIM algebra, there is an elliptic algebra $U_{q,p}(\widehat{\mathfrak{g}})$ which has close connection with the q -Virasoro/ q -W algebra. This is a face-type elliptic algebra constructed based on an elliptic solution to the dynamical Yang-Baxter equation. This algebra also has the Hopf algebra structure. We expect that this algebra plays the role of the “dynamical symmetries” of the 5d gauge theories. Recently, we have studied correspondence between a level-1 $U_{q,p}(\widehat{sl}(2))$ module and 5d $SU(2)$ gauge theory. We will generalize this result to more general elliptic algebra $U_{q,p}(\widehat{\mathfrak{g}})$.