Results of research

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We observe that the Dotsenko-Fateev integral representation of the conformal block of 2d conformal field theory can be interpreted as a β-deformed matrix model of Selberg type. Using the formula associated with the Jack polynomials, we established the method of generating q-expansion coefficients for conformal block and Nekrasov function for $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with four flavors (Ref. [28] of the Publication List).

We then consider a series of massive scaling limits of the β-deformed matrix model of Selberg type ($SU(2)$ with $N_f = 4$) which reduce the number of flavors to $N_f = 3$ and subsequently to $N_f = 2$ ([29]).

We consider β-deformed quiver matrix model based on the affine Lie algebra $A_n^{(1)}$. The Virasoro constraint of this model is determined. For $n = 1, 2$ cases, the explicit forms of the loop equations are obtained ([30]).

The (W)AGT conjecture implies there is a correspondence between the partition functions of the four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories and the conformal blocks of the two-dimensional theories with the Virasoro or W symmetries. The "q-deformed" version of (W)AGT conjecture states that the q-lifted version of the partition function of five-dimensional gauge theories and the "conformal blocks" of the two-dimensional theories with the q-deformed Virasoro/W symmetries. Starting from this q-version of (W)AGT conjecture, we demonstrate by taking a certain r-th root of unity limit in q, the correspondence between the four-dimensional partition function on the ALE space of A-type and the conformal blocks of the two-dimensional theories with the super-Virasoro symmetry or its generalization is automatically generated ([32]). Furthermore, we demonstrated that the parafermions appear in the r-th root of limit of the q-deformed Virasoro and the q-deformed W algebra ([34]).

A q-deformed vertex operator is determined from the five-dimensional $SU(2)$ Nekrasov partition function based on the q-AGT conjecture. We obtained a q-deformed version of Coulomb gas representation of the conformal block by using the vertex operators and q-screening charges. After slightly changing the position of one of the vertex operators, we have checked that the q-block coincides with 5D Nekrasov function in low degrees of instanton expansion ([35]).

A set of Schwinger-Dyson equations for the resolvents are considered in a class of supersymmetric Chern-Simons-matter matrix models. In the planar limit, these loop equations reduce to two independent algebraic cubic equations for the two planar resolvents ([36]).

We argue that the level-1 elliptic algebra $U_{q,p}(\widehat{\mathfrak{g}})$ is a dynamical symmetry in the correspondence between 2d field theories and 5d supersymmetric gauge theories. A level-1 $U_{q,p}(\widehat{\mathfrak{sl}}(2))$ module can be realized by an elliptic vertion of the Frenkel-Kac construction. In a r-th root of unity limit of the deformation parameter p, the $\mathbb{Z}_r$-parafermions and a free boson appear. And the 2d/5d correspondence goes to the correspondence between the 2d coset CFT with para-Virasoro symmetry and 4d $\mathcal{N} = 2$ $SU(2)$ gauge theory on $\mathbb{R}^4/\mathbb{Z}_r$.