# Research Result

Nobuhiko Tahara

Painlevé equations are six nonlinear ordinary differential equations of second order which were obtained in the early 20th century by Painlevé and his student from the study of new special functions, not reducible to classical special functions, defined by algebraic ordinary differential equations with no movable singularities other than poles. They are related to isomonodromic deformations of second order Fuchsian differential equation, 2D Ising model, and other research areas. Okamoto constructed the space of initial conditions for each Painlevé equations which contains of all solution curves of it as isomonodromic flow in the fiber space. Each of them is obtained by blowing-up the singularities of the compactification of the phase space of the Hamiltonian system equivalent to a Painlevé equation and removing several vertical leafs. They characterize Painlevé equations from geometric view. Okamoto further showed that existence of Bäcklund transformation for each Painlevé equations (except the first) which acts as affine Weyl group to the parameter space of the equation.

Noumi and Yamada proposed the system with affine Weyl group symmetry of type $A_\ell^{(1)}$ for any $\ell = 2, 3, \ldots$, as a the higher order generalization of Painlevé equations, from the research of generalization of the correspondence of affine root system and Bäcklund transformation of nonlinear differential equation.

I have interested to the relationship between differential equations and such manifolds and to the natural symmetry with Painlevé equations, and obtained the augmented phase space for the system of affine Weyl group symmetry of type $A_4^{(1)}$ which parametrizes all meromorphic solutions of it. While the phase space for each Painlevé system is two-dimensional, the phase space in our case is four-dimensional and the calculations in blowings-up are highly complicated. By the approach based on singularity analysis, we construct the augmented phase space consisting of the original phase space, codimension 1 spaces and codimension 2 spaces.