

## Summary of my research

In my master thesis, I studied a decomposition theory of exponential operators and proposed [1]<sup>1</sup> a general scheme to construct independent determining equations for relevant decomposition parameters by using Lyndon words on a free Lie algebra.

In my Ph.D. thesis, I studied an analytic Bethe ansatz (ABA) and  $T$ -systems (a system of functional relations among transfer matrices of solvable lattice models in statistical mechanics). We gave solutions of the  $T$ -system for  $U_q(D_r^{(1)})$  [2] and discretized affine Toda field equations [3]. I proposed [4-8] a general theory on ABA based on the Bethe ansatz equations associated with superalgebras. Based on the  $T$ -system proposed in [8], we derived [9-11,15] TBA equations (nonlinear integral equations for the free energy of quantum integrable systems at finite temperature).

We studied fermionic formulae and  $Q$ -systems related to completeness of the Bethe ansatz [12-14]. I also constructed [16] a Casorati determinant solution of a  $T$ -system.

In general, TBA equations is an infinite number of coupled nonlinear integral equations (NLIE) with an infinite number of unknown functions. Then I systematically derived NLIE with a finite number (= rank) of unknown functions, which are equivalent to the TBA equations, associated with Lie algebras of arbitrary rank [17,18,20,21,23]. They are applicable to calculations on thermodynamics of spin ladder systems. We had an excellent agreement with experimental results [19,25].

Evaluation of correlation functions in the statistical mechanics is a very difficult problem even for solvable lattice models. We succeeded to evaluate correlation functions at finite temperatures based on the high temperature expansion technique [22,24].

We constructed [26] Baxter  $Q$ -operators for  $U_q(\hat{sl}(2|1))$  based on infinite dimensional representations of  $q$ -oscillator algebras. Our formulation can be applicable to both lattice models and the CFT.

I proposed [27] Wronskian-type solutions of the  $T$ -system for  $U_q(\hat{gl}(M|N))$ . I identified that the number of the Baxter  $Q$ -operators is  $2^{M+N}$  for the first time. Based on techniques and ideas proposed in [27], we gave solutions of the  $T$ -,  $Q$ - and  $Y$ -system for the AdS/CFT [28,29,31].

We proposed [30] a new definition of Baxter  $Q$ -operators based on the ‘co-derivative’ on supercharacters of  $gl(M|N)$ , and derived Bethe ansatz equations without using the Bethe ansatz.

We proposed [32,33] the ‘master  $T$ -operator’, which is a sort of generating function for transfer matrices for any generalized quantum integrable spin chain. It corresponds to a  $\tau$ -function of classical integrable hierarchies of soliton equations. Baxter  $Q$ -operators and functional relations in Hirota bi-linear form are systematically derived from it.

Based on properties of the universal  $R$ -matrix, we proved [34] the fact that  $L$ -operators for Verma modules for  $U_q(\hat{sl}(2))$  factorize with respect to  $L$ -operators for Baxter  $Q$ -operators.

I studied asymptotic representations for the quantum affine superalgebra  $U_q(\hat{gl}(M|N))$  and obtained solutions ( $L$ -operators) of the graded-Yang-Baxter equation related to Baxter  $Q$ -operators [35].

We proposed [37] a decomposition formula of Beisert’s  $S$ -matrix for the AdS/CFT with respect to the  $R$ -matrix of the free fermion model and pointed out a relation to  $U_q(\hat{sl}(2))$  at  $q = i$ .

In [36], we extended results in [32] to  $gl(N|M)$  case. The zeros of the eigenvalues of the master  $T$ -operator obey an equation of motion for the Ruijsenaars-Schneider model. We also proposed a set of algebraic equations, which gives the eigenvalues of Hamiltonians for supersymmetric spin chains.

We solved [39] intertwining relations of the augmented  $q$ -Onsager algebra, and obtained generic boundary  $K$ -operators in terms of the Cartan element of  $U_q(sl_2)$ . Considering asymptotic representations of the algebra in Verma modules, we derived  $K$ -operators for Baxter  $Q$ -operators for integrable models with open boundary conditions.

I gave [38] a solution of the set-theoretic (quantum) Yang-Baxter equation as a product of quasi-Plücker coordinates (quasi-determinant) over a matrix composed of  $L$ -operators (which are images of the universal  $R$ -matrix for  $U_q(gl(n))$ ). In the quasi-classical limit, this reduces to a new determinant (Plücker coordinate) expression of a classical Yang-Baxter map.

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<sup>1</sup>See my publication list for the reference number of each paper.