Research Plan

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I. Strongly quasi-hereditary endomorphism algebras.

Quasi-hereditary algebras were introduced by Cline, Parshall and Scott to study highest weight categories which arise in the representation theory of semisimple complex Lie algebras and algebraic groups. Quasi-hereditary algebras were widely studied by Dlab and Ringel from the viewpoint of the representation theory of algebras. Motivated by Iyama's finiteness theorem of representation dimensions of artin algebras, Ringel introduced the notion of right-strongly quasi-hereditary algebras. In the representation theory of algebras, right-strongly quasi-hereditary algebras are frequently realized as endomorphism algebras. For example, endomorphism algebras of certain cluster tilting modules over a preprojective algebra, Auslander algebras and Auslander–Dlab–Ringel (for short ADR) algebras are right-strongly quasi-hereditary endomorphism algebras.

One of aims of this research plan is to give a new construction of right-strongly quasi-hereditary endomorphism algebras, containing such examples. Recently, Coulembier introduced a new class of artin algebras which is a generalization of ADR algebras, and gave a sufficient condition for his algebra to be quasi-hereditary. I showed that, under certain conditions, his algebras can be realized as endomorphism algebras. It is natural to ask the following question.

Question 1. When are Coulembier's endomorphism algebras right-strongly quasi-hereditary?

I will give a complete answer of this question and show that the endomorphism algebras of certain cluster tilting modules over a preprojective algebra and Auslander algebras are given by Coulembier's endomorphism algebras.

II. Koszulness of Coulembier's endomorphism algebras.

My aim of this study is to give a criterion for Coulembier's endomorphism algebras to be Koszul algebras. I gave a characterization of ADR algebras to be strongly quasi-hereditary. Moreover, I gave an equivalence condition that strongly quasi-hereditary ADR algebras are Koszul (this is joint work with T. Adachi and A. Chan). Since Coulembier's endomorphism algebras are a generalization of ADR algebras, I will study Koszulness of strongly quasi-hereditary Coulembier's endomorphism algebras. When Coulembier's quasi-hereditary endomorphism algebras are Koszul, there exist two duals: Ringel dual (given by quasi-hereditary algebra structures) and Koszul dual (given by Koszul algebra structures). I pose the following question.

Question 2. Does Ringel dual and Koszul dual commute to each other for given Coulembier's quasi-hereditary endomorphism algebra?

Mazorchuk showed that if a graded quasi-hereditary algebra is balanced, then it is Koszul. Moreover, its Ringel dual and its Koszul dual commute. I expect to give a criterion for Coulembier's quasi-hereditary endomorphism algebra to be balanced.

III. Rejective chains of τ-categories.

I characterized rejective chains of the category of finitely generated modules over a representation-finite artin algebra and the category of finitely generated projective modules over an artin algebra respectively. The category of finitely generated modules over an artin algebra is a τ-category which was introduced by Iyama to study the category of lattices over orders. The categorical structures of τ-categories are given by Auslander–Reiten quivers. From the viewpoint of combinatorics, I will study rejective chains of τ-categories. As an application, I expect to give a characterization of relative Auslander algebras to be strongly quasi-hereditary. This is a generalization of III in Research Result.