Research Result

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I. Hochschild cohomology of q-Schur algebras.

The notion of q-Schur algebras were introduced by Dipper and James in order to study the modular representation theory of finite general linear groups. It is known that q-Schur algebras over a field are quasi-hereditary. Hochschild cohomology of associative algebras appears in many areas of mathematics. One of the most important properties of Hochschild cohomology is its invariance under derived equivalences.

To compute the Hochschild cohomology of any block of q-Schur algebras, I proved the following results. First, I constructed a derived equivalence between any block of a q-Schur algebra and a "special" block of some q-Schur algebra. Since the "special" block is explicitly given by a bounded quiver algebra, one can easily calculate the Hochschild cohomology. Second, I determined some part of the ring structure of the Hochschild cohomology ring by using quasi-hereditary algebra structures of q-Schur algebras.

II. Characterizing strongly quasi-hereditary algebras.

Motivated by Iyama's finiteness theorem of representation dimensions of artin algebras, Ringel introduced the notion of right-strongly (resp. left-strongly) quasi-hereditary algebras from the viewpoint of highest weight categories. It is known that right-strongly quasi-hereditary algebras have better upper bound of global dimension than that of ordinary quasi-hereditary algebras.

I gave characterizations of these algebras in terms of right-strongly heredity chains, total right rejective chains and coreflective chains. Moreover, I gave a characterization of strongly quasi-hereditary algebras (they are a special class of right-strongly/left-strongly quasi-hereditary algebras) by using rejective chains. One of the advantages of these results is that one can give a right-strongly quasi-hereditary algebra structure without a quasi-hereditary algebra structure. For example, I obtained that locally hereditary algebras and Nakayama algebras which have heredity ideals are right-strongly quasi-hereditary by applying my result.

As an application, I refined a well-known result of Dlab–Ringel stating that any artin algebra of global dimension at most two is quasi-hereditary. Namely, I proved that such an algebra is always right-strongly quasi-hereditary.

III. Characterizing strongly quasi-hereditary Auslander algebras.

Since the global dimension of any Auslander algebra is at most two, it is right-strongly quasi-hereditary by the result II. However, Auslander algebras are not necessarily strongly quasi-hereditary. Applying my results of rejective chains, I gave the following characterization of Auslander algebras to be strongly quasi-hereditary.

Let A be a representation-finite artin algebra and B the Auslander algebra of A. Then B is a strongly quasi-hereditary algebra if and only if A is a Nakayama algebra.

IV. On upper bound for global dimension of Auslander–Dlab–Ringel algebras.

Let A be an artin algebra with Loewy length m and J the Jacobson radical of A. Auslander studied the endomorphism algebra $B := \mathrm{End}_A(\bigoplus_{j=1}^{m} A/J^j)$ and proved that B has finite global dimension. Furthermore, Dlab and Ringel showed that B is quasi-hereditary. Hence B is called an Auslander–Dlab–Ringel (ADR) algebra. Recently, Conde gave a left-strongly quasi-hereditary algebra structure on ADR algebras.

I studied ADR algebras of semilocal modules introduced by Lin and Xi. Since any artin algebra is a semilocal module, the ADR algebras of semilocal modules are a generalization of the original ADR algebras. I proved that ADR algebras of semilocal modules are left-strongly quasi-hereditary. As an application, I gave a tightly upper bound for global dimension of an ADR algebra.