I will develop my research further. In particular, I will study about the 2d-4d(5d,6d) connection.

2d/4d(5d,6d) connection

I will study the AGT relation which states that there exists correspondence between two-dimensional conformal field theory (CFT) and four-dimensional supersymmetric gauge theory. This correspondence is lifted to the 2d/5d correspondence by q-deformation. The original 2d/4d correspondence can be reproduced in the limit $q \to 1$.

Since the q-Virasoro/W_N algebra appears in the level N representation of Ding-Iohara-Miki (DIM) algebra, it is considered that the DIM algebra plays an important role behind the 2d/5d connection. The DIM algebra is a kinds of q-deformation of $W_{1+\infty}$ algebra. I will construct the families of matrix models which have the constraints for $W_{1+\infty}$ algebra as the Schwinger-Dyson equations. Based on these research, I will generalize to the DIM algebra. It is expected that the interesting models, such as the Chern-Simons matrix model and ABJM matrix model, belong to the class satisfying the constraints for the DIM algebra. In addition, I will consider its correspondence to the gauge theory.

So far I succeeded to construct the integral representation of the $\mathfrak{sl}(2)$ current blocks with K operator inserted, which correspond to the SU(2) instanton partition function of the supersymmetric gauge theories in the presence of surface operators. I would like to generalize to the case of SU(N) and $\mathfrak{sl}(N)$. Moreover, it is not clear what role the pure current blocks without K operator have in the gauge theory. I will try to this problem.

The 2d/6d correspondence is obtained by elliptic deformation. Then the elliptic Virasoro algebra appears. Based on the previous works on 2d/4d(5d) correspondence, I will study the effects of the elliptic deformation.

In a classical limit of the theory of the both side in 2d-4d connection, two different integrable models, the Gaudin model from 2d side(CFT) and the Heisenberg model from 4d side(gauge theory), are obtained. Therefore the 2d-4d connection suggests that there exists a relation between these integrable models. I would like to consider the classical limit of the parafermionic coset CFT and the supersymmetric gauge theory in ALE spacetime and study the corresponding integrable model.