Conference on
Automorphic Forms and Automorphic L-Functions
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Research Institute for Mathematical Sciences
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ABSTRACTS

Jan Hendrik Bruinier (Universität zu Köln)

Traces of CM-values of modular functions

Abstract: Zagier proved that the traces of singular moduli, i.e., the sums of the values of the classical j-invariant over quadratic irrationalities, are the Fourier coefficients of a modular form of weight 3/2 with poles at the cusps. Using the theta correspondence, we generalize this result to traces of CM values of (weakly holomorphic) modular functions on modular curves of arbitrary genus. We also discuss some further generalizations. (Joint work with J. Funke.)

Hilbert class polynomials and traces of singular moduli

Abstract: We prove new explicit formulas for the traces of singular moduli. They can be used to compute certain limits involving the values of j at CM points of discriminant d as d goes to infinity. (Joint work with P. Jenkins and K. Ono.)

Yoshiki Hayashi

Correspondences between the spaces of Eisenstein series of Jacobi forms and modular forms with primitive character

Abstract: For an odd prime p we construct the lifting from the spaces of Eisenstein series $J_{k,m}^{\mathrm{Eis}}(\Gamma_0(p), \chi_p)$ to a certain subspace of $M_{2k-2}^{\mathrm{Eis}}(mp)$ where χ_p a (real) primitive Dirichlet character modulo p:

In order to construct this lifting, we consider followings:

1. Definition of the Jacobi form $\phi_{k,m}$ of weight k and index m on $\Gamma_0(p) \ltimes \mathbb{Z}^2$ with primitive character χ_p.

2. Construction of the Jacobi Eisenstein series $E_{k,m,\chi_p}^{\kappa}(\tau, z) \in J_{k,m}^{\mathrm{Eis}}(\Gamma_0(p), \chi_p)$ (κ: cusps $\infty, 0$).

3. Explicit formula of the Fourier coefficients of $E_{k,1,\chi_p}^{\infty}(\tau, z)$.

4. Construction of the general Jacobi Eisenstein series in $J_{k,m}^{\mathrm{Eis}}(\Gamma_0(p), \chi_p)$ as linear sum of E_{k,m,χ_p}^{κ}.

4. Hecke operator and Atkin-Lehner involution on the space $J_{k,m}^{\mathrm{Eis}}(\Gamma_0(p), \chi_p)$.

5. Construction of correspondence between the space $J_{k,m}^{\mathrm{Eis}}(\Gamma_0(p), \chi_p)$ and a certain subspace of $M_{2k-2}^{\mathrm{Eis}}(mp)$.

Yumiko Hironaka (Waseda University)

Spherical functions of certain p-adic homogeneous spaces

Abstract: Let $\mathbb{G}$ be an algebraic group defined over a $\mathfrak{p}$-adic field k and consider the set of k-rational points X of a suitable $\mathbb{G}$-homogeneous space. A nonzero K-invariant function on X is called *a spherical function on X* if it is a common eigenfunction with respect to the action of Hecke algebra $\mathcal{H}(G, K)$, where K is a compact open subgroup of $G = \mathbb{G}(k)$.

Spherical functions are a basic tool to inverstigate G-space X, and in some cases they have a close relation to classical number theory, e.g. they can be regarded as generating functions of local densities of symmetric forms.

In the talk, she will introduce a mechanism of those functional equations for certain spherical homogeneous spaces.

Atsushi Ichino (Osaka City University)

On critical values of adjoint L-functions for GSp(4)

Abstract: We discuss Deligne's conjecture on critical values of adjoint L-functions for GSp(4). As an evidence for our speculation, we give a formula for Petersson norms of certain non-holomorphic automorphic forms on GSp(4) in terms of critical values.

Yoshi-hiro Ishikawa (Okayama University)

On standard L-function for generic cusp forms on U(2, 1)

Abstract: We define ramified factors of the standard L-function for U(2, 1) through the Rankin-Selberg integral and show the global functional equation when the cusp form is globaly generic. Over non-archimedean places, we would like to compare the L-factors defined by other methods.

Dihua Jiang (University of Minnesota)

Non-vanishing of the central value of the Rankin-Selberg L-function I, II

Abstract: Following the recent progress on the Langlands functorial lifting from the classical groups to the general linear group, Lapid and Rallis proved the central value of the Rankin-Selberg L-functions of symplectic type is non-negative. I am going to report on the joint work with Ginzburg and Rallis of the characterization of the nonvanishing of the central value of the Rankin-Selberg L-functions in terms of periods of automorphic forms over classical groups. This generalizes the Gross-Prasad conjecture for orthogonal groups and give a symplectic version of the Gross-Prasad conjecture. The unitary group version of this work is the joint work in prograss with Ginzburg.

Erez Lapid (Hebrew University)

The relative trace formula and its applications I, II

Abstract: In the first talk I will review the evolution of the relative trace formula and various contexts it was applied. In the second talk I will focus on more recent developments and future directions.

Wenzhi Luo (Ohio State University)

Equidistribution of Hecke eigenforms on arithmetic surfaces I, II

Abstract: In these two talks, we will survey the recent progress on the equidistribution of the Hecke eigenforms on arithmetic hyperbolic surfaces, which is analogue of the ergodicity of Laplacian square-integrable eigenfunctions on surfaces whose geodesic flows are ergodic, as well as the high-dimensional generalization to holomorphic sections of high powers of an ample Hermitian line bundle over a Kähler manifold. In particular, we'll give applications of a remarkable relation (à la Jacquet, Harris-Kudla and Watson) between the equidistribution of eigenforms and the degree 8 triple product L-functions. The two talks are mostly based upon my recent joint works with Peter Sarnak.

Kohji Matsumoto (Nagoya University)

The mean square of standard L-functions attached to Ikeda lifts

Abstract: Let $L(s, F)$ be the standard L-function attached to a Siegel cusp form F. When F is an Ikeda lift, one can obtain rather sharp estimates of the mean square of $L(s, F)$ with respect to $\text{Im}(s)$. In this talk I report such estimats, a part of which is a joint work with A. Sankaranarayanan.

Philippe Michel (Université de Montpellier II)

Some Applications of Kloosterman sums I *(algebraic theory),* II *(spectral theory)*

Abstract: For more than 60 years, Kloosterman sums have been a central tool for analytic number theory and in particular for the analytic theory of modular forms and their associated L-functions; even now the flow of applications coming for Kloostermania is not dried up. In these talks we will review a few of them.

On the one hand, Kloosterman sums are algebraic exponential sums and in fact (more importantly for us) they can be realized as the traces of Frobenius of some ℓ-adic sheaf; in the first talk we will describe some results combining this ℓ-adic aspect with methods from classical analytic number theory.

On the other hand, as is well known, Kloosterman sums are strongly linked with the theory of modular forms via the Petersson/Kuznetzov formula. In the second talk, we explain some recent applications of this connection (like subconvexity bounds for L-function) and also some other applications obtained by mixing altogether the ℓ-adic and modular aspects of Kloosterman sums.

Stephen D. Miller (Hebrew University and Rutgers Univeersity)

On the analytic continuation of the exterior square L-function I, II

Abstract: Both integral representations and the Langlands-Shahidi method give detailed information about the analytic properties of various automorphic L-functions. However, in most cases this information is unfortunately not complete in practice. For example, integral representations are often produced which are entire, but which are not completely identified with their targeted Langlands L-function. On the other hand, the Langlands-Shahidi method produces the desired functional equation, but in some cases does not rule out having unwanted poles. We have pursued a third method, that of pairing automorphic distributions over flag varieties. It is formally similar to the method of integral representations, but instead derives holomorphy from oscillatory properties of distributions rather than the rapid decay of cusp forms. As a test case the method gives new examples of the full holomorphic continuation of the exterior square L-functions on $\text{GL}(n)$ that were not obtained by the previous methods. The talk will focus on the notion of automorphic distribution and these examples of the exterior square. (Joint work with Wilfried Schmid)

Hiro-aki Narita (Kyoto Sangyo University)

Explicit construction of automorphic forms on $\mathrm{Sp}(1,q)$

Abstract: There are three typical constructions of automorphic forms: Eisenstein series, Poincare series and theta series. We carry out these constructions for automorphic forms on $\mathrm{Sp}(1,q)$ generating quaternionic discrete series in the sense of Gross-Wallach. As for the construction by theta series, we think of the theta lifting from elliptic cusp forms to automorphic forms on $\mathrm{Sp}(1,q)$, formulated by T. Arakawa. We shall prove that the images of the Arakawa's lifting are the automorphic forms on $\mathrm{Sp}(1,q)$ just mentioned above.

Chris Skinner (University of Michigan)

Main conjectures and modular forms I, II

Abstract: In these talks I will report on work relating the p-adic L-function of an eigenform (for $\mathrm{GL}(2)$) that is ordinary at the prime p to the characteristic polynomial of the eigenform's associated p-adic Selmer group. The Main Conjecture of Iwasawa Theory for eigenforms asserts the equality of the ideals generated by the p-adic L-function and the characteristic polynomial, respectively. K. Kato has shown that one has that the characteristic polynomial divides the p-adic L-function. I will discuss recent work, joint with E. Urban, that essentially establishes the opposite divisibility in many cases (and so the full conjecture in combination with Kato's results). This work makes use of congruences between Eisenstein series and cuspforms on the unitary groups $\mathrm{U}(2,2)$ and of the Galois representations assiated to certain automorphic representations of unitary groups.

Endoscopic/stable congruences for unitary groups

Abstract: I will discuss joint work with M. Harris and J-S. Li the goal of which is to

(1) construct general p-adic L-functions for unitary groups and

(2) relate congruences between stable forms on $\mathrm{U}(n+1)$ and endoscopic lifts from $\mathrm{U}(n)$ to values of (p-adic) L-functions.

This work makes use of the doubling method, explicit theta lifts and Siegel-Weil formulas, and the Rallis inner-product formula.

Masao Tsuzuki (Sophia University)

Automorphic Green currents on $\mathrm{U}(n,1)$

Abstract: We will explain an explicit way to construct a Green current for a modular cycle in an arithmetic quotient of the unitary group $\mathrm{U}(n,1)$. We shall also compute a Fourier coefficient of automorphic Green function (at a character of maximal unipotent subgroup of $\mathrm{U}(n,1)$) to have an identity between a certain average of Fourier coefficients of elliptic modular forms and a certain average of period integrals of modular forms on $\mathrm{U}(n,1)$.

Satoshi Wakatsuki (Osaka University)

An explicit dimension formula for the spaces of vector valued Siegel cusp forms of degree two

Abstract: We show an explicit dimension formula for the spaces of vector valued Siegel cusp forms of degree two. Tsushima already gave the dimension formula by using the Riemann-Roch theorem. We prove the dimension formula by using the Selberg trace formula and the theory of prehomogeneous vector spaces. In addition to the general theory, we need some calculation techniques to calculate explicitly the dimension formula. One of our motivation is as follows. Ibukiyama gave a conjecture for the Shimura correspondence of vector valued Siegel cusp forms of degree two. In order to prove this conjecture, we must show the equality of the traces of Hecke operators. As the first step, we treat the traces of the trivial actions, which are the dimensions of the spaces.