Braid index of spatial graphs

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0 Spatial Graph

In this paper we work in piecewise linear category. Let G be a finite graph, and $\mathbb{R}^3$ be an 3-dimensional Euclidean space. A *spatial embedding* of G is an embedding $g : G \to \mathbb{R}^3$ of G, and its image is called a *spatial graph*. A graph G is *planar* is there exists a embedding $G \to \mathbb{R}^2$. A *diagram* is a regular projection of spatial graph that has relative height information added to it at each of the double points.

1 Reidemeister moves for Spatial Graphs

Theorem 1.1 *Two spatial embeddings f and g is ambient isotopic if there exists an orientation preserving homeomorphism Φ such that $\Phi \circ f = g$.*

Kauffman has defined *Reidemeister moves for graphs*[1], which consist of traditional Reidemeister moves for links and extra two moves involving a vertex.

Theorem 1.2 *If two spatial graphs are ambient isotopic then any two diagrams of them are related by a finite sequence of Reidemeister moves for graphs.*

2 Braid index for θ_n-curve

Let G be a θ_n-curve in $\mathbb{R}^3$, such that all edges are oriented so that the origin and terminus of each edges are the same. We can obtain a *braid presentation* of θ_n-curve in the way that we obtained a link by a closure of a braid[2].

Theorem 2.1 *Any θ_n-curve has a braid presentation.*

We intend to expand this braid presentation for simply oriented θ_n-curve to an arbitrarily oriented θ_n-curve and other spatial graphs.

References

[1] L.Kauffman, *Invariants of graphs in three-space*, Trans. Amer. Math. Soc., **311**(1989), 697-710.

[2] T.Shinnoki, and T.Takamuki, *On the braid index of θ_m-curve in 3-space*, Math. Nachr., **260**(2003), 84-92.