NZ-Japan Knot Theory Conference Abstracts

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Comparison of end invariant and Jorgensen's side parameter

Hirotaka Akiyoshi
(Osaka City University Advanced Mathematical Institute)

Abstract

A punctured torus group is a Kleinian group freely generated by two isometries whose commutator is parabolic. By the positive answer to Thurston's ending lamination conjecture for punctured torus groups due to Minsky, such groups are parametrized by end invariant and every such group is obtained as an algebraic limit of quasifuchsian punctured torus groups. Thus, by Jorgensen's theory on the combinatorial structures of Ford domains, one can define the side parameter on the space of punctured torus groups. In this talk, we compare the two parameters and show that the end invariant and the side parameter are in a uniformly bounded Weil-Petersson distance.

Fixed subgroups in free groups

Laura Ciobanu (University of Auckland)

Abstract

I will give an overview of the area of fixed subgroups in free groups, an area in which most results have been proved topological and geometric methods. Then I will present joint work with Warren Dicks. We show there exist subgroups that are equal to the fixed subgroup of some set of endomorphisms but are not equal to the fixed subgroup of any set of automorphisms. Moreover, we determine the Galois monoids of the retracts we study, where, by the Galois monoid of a subgroup H of F, we mean the monoid consisting of all endomorphisms of F that fix H.

Unkottedness of short geodesics in hyperbolic 3-manifolds

Richard Evans (University of Auckland)

Abstract

Recent new methods introduced into the study of the deformation spaces of hyperbolic 3-manifolds of infinite volume by Bromberg exploit properties of short geodesics. The key property, due to Otal, Souto, is that short geodesics in hyperbolic 3-manifolds of inifinte volume are unknotted. We discuss this result, how unknottedness is used by Bromberg and finish by briefly discussing some of the applications of Bromberg's work.

Weyl Algebras and knot invariants

Roger Fenn (Sussex University)

Abstract

The Weyl algebra, $uv - vu = 1$, and the quantum Weyl algebra, $uv - qvu = 1$, can be used to give new knot on surface invariants. This is work in progress with V. Turaev.

Long Line Knots

David Gauld (University of Auckland)

Abstract

Classical Knot Theory has knotted copies of the real line in 3-space or, to avoid unintended untying, knotted circles. There are other possible 1-manifolds that one might consider knotting in 3-space. The most likely is the long line, but to embed it at all the ambient space must be long enough to include the long ends: a power of the long line will do. Interestingly one does not have to worry about inadvertent untying. I shall discuss this as well as related placement problems involving non-metrisable manifolds.

Seifert surfaces in open books and a new coding algorithm for links

Mikami Hirasawa (Gakushuin Univ.)

Abstract

This is a joint work with T. Kobayashi (Nara Women's Univ) and Rei Furihata (Yosami Junior High School) We introduce a new standard form of a Seifert surface F. In that standard form, F is obtained by successively plumbing flat annuli to a disk D, where the gluing regions are all in D. We show that any link has a Seifert surface in the standard form, and thereby present a new way of coding a link. We present an algorithm to read the code directly from a braid-presentation.

On holomorphic sections of holomorphic families of Riemann surfaces

Yoichi Imayoshi (Osaka City University)
Toshihiro Nogi (Osaka City University)

Abstract

We consider the Diophantine problem over a field of algebraic functions of one variable. For a closed Riemann surface $\bar{R}$, we study a Diophantine equation

$$\sum_{i+j+k=n} A_{ijk} X^i Y^j Z^k = 0,$$

where the coefficients A_{ijk} and the unknowns X, Y, Z are all meromorphic functions on $\bar{R}$.

For this problem we have the following equivalent geometric formulation: We may assume there exists a finite subset B of $\bar{R}$ such that for $^{\forall} t \in R = \bar{R} \setminus B$ the algebraic curve S_t defined by

$$S_t = \{[x, y, z] \in P^2(\mathbf{C}) \mid \sum_{j+j+k=n} A_{ijk}(t) x^i y^j z^k = 0\}$$

is a Riemann surface of fixed genus g. We set

$$M = \bigsqcup_{t \in R} \{t\} \times S_t,$$

$$\pi \colon M \to R, \pi(t, p) = t.$$

Then (M, π, R) becomes a holomorphic family of closed Riemann surfaces of genus g over R, and a solution (X, Y, Z) gives rise to a holomorphic section s of (M, π, R) given by $s(t) = (t, [X(t), Y(t), Z(t)])$.

By using theory of Teichmüller space, Imayoshi and Shiga gave a proof for a finiteness theorem of sections (Mordell conjecture) and a finiteness theorem of families (Shafarevich conjecture).

In this talk we deal with a very concrete holomorphic family (M, π, R) of closed Riemann surfaces of genus two over a fourth punctured torus R, which is a kind of a Kodaira surface and is constructed by Riera. We give an explicit defining equation of (M, π, R) by using elliptic functions, and determine all the holomorphic sections of (M, π, R).

The virtual magnetic skein module and construction of skein relations on Jones–Kauffman polynomial

Atsushi Ishii (Osaka University),
Naoko Kamada (Osaka City University) and
Seiichi Kamada (Hiroshima University)

Abstract

The Jones polynomial of a classical link is regarded as normalized bracket polynomial due to Kauffman. It is also defined for a virtual or virtual magnetic link, which we call the Jones–Kauffman polynomial or the f-polynomial.

Let L_p, L_n, L_0, L_∞ and L_v be virtual or virtual magnetic links whose diagrams are the same outside a disk region where they have a positive crossing, a negative crossing, 0-smoothed one, ∞-smoothed one, and a virtual crossing, respectively. The f-polynomials of these links are not independent. 'The skein relation' (in the classical sense) means a relation among f-polynomials of three links L_p, L_n and L_0. A virtual skein relation is a relation among f-polynomials of L_p, L_n and L_v. N. Kamada, S. Nakabo and S. Satoh studied about such a relation. A magnetic skein relation is a relation among f-polynomials of L_0, L_∞ and L_v.

We introduce the notion of virtual magnetic skein module, and using it, we give a systematic method to construct various virtual and magnetic skein relations, including known ones.

The classification of closed 2-string virtual braids

Teruhisa Kadokami
(Osaka City University Advanced Mathematical Institute)

Abstract

Let L and L' be two virtual links. We would like to consider whether L is equivalent to L' or not. We call the problem "Detecting Problem".

For closed 2-string virtual braids, we can classify them completely. Let $K = K_{p_1,\cdots,p_n}$ and $K' = K_{p'_1,\cdots,p'_{n'}}$ be two closed 2-string virtual braids (We explain concretely the meaning of the notations in the talk). We show that K and K' are equivalent if and only if (1) $n = n'$, and (2) $p'_i = p_{\sigma(i)}$ where $\sigma(i) = i + k \pmod n$ for some integer k, are satisfied.

A Skein Relation for the HOMFLYPT Polynomials of Two-Cable Links

Taizo Kanenobu (Osaka City University)

Abstract

We give a skein relation for the HOMFLYPT polynomials of 2-cable links. In [A. Ishii and T. Kanenobu, *Different links with the same Links-Gould invariant*, Osaka J. Math. **42** (2005) 273–290], we construct examples of arbitrarily many 2-bridge knots sharing the same HOMFLYPT, Kauffman, and Links-Gould polynomials, and arbitrarily many 2-bridge links sharing the same HOMFLYPT, Kauffman, Links-Gould, and 2-variable Alexander polynomials. Using the skein relation, we show their 2-cables also share the same HOMFLYPT polynomials.

Various Bennequin inequalities on Knot Concordance invariants

Tomomi Kawamura (Aoyama Gakuen University)

Abstract

We estimate the Rasmussen invariant, the Ozsváth-Szabó τ-invariant, and other knot concordance invariants. Furthermore, we improve the slice-Bennequin inequality on knots, if we can.

Grading the surface-link groups

Akio Kawauchi (Osaka City University)

Abstract

We know that the set of the groups of n-dimensional manifold-links for $n > 2$ coincides with the set SLG of the groups of surface-links. Let $SLG(r;A)_g$ be the subset of SLG consisting of the groups of surface-links with component number r and total genus g whose second homology of the group is isomorphic to A. We show that the set $SLG(r;A)_g$ is a non-empty proper subset of $SLG(r;A)_{g+1}$ for every g and every A generated by $2g$ elements. We also determine the sets $SLG(r;A)_g$ to which every given classical link group belongs.

Quasiregular mappings, knotted branch sets and hyperbolic groups

Gaven Martin (Massey University)

Abstract

The generalised Lichnerowicz conjecture asks us to classify those compact n-manifolds which admit a rational endomorphism (or uniformly quasiregular mapping). For space forms this has been done by Mayer, Martin and Peltonen using, in the hyperbolic case, quite deep results of Gromov, Varopoulous et al. They showed a much stronger result in the case of zero curvature - there are no branched quasiregular self mappings of flat manifolds whatsoever. In this talk we show the same is true in the negatively curved case and in somewhat more generality. This builds on the deep work of Sela concerning the Hopfian and co-Hopfian properties of hyperbolic groups and some nice work of Walsh on open mappings between manifolds. Generalisations of this result to the class of relatively hyperbolic groups apply to hyperbolic knot complements and can be used to show that these knots cannot be the branch sets of discrete open maps.

Deformation of abelian representations and twisted Alexander invariants

Takayuki Morifuji (Tokyo University of Agriculture and Technology)

Abstract

In this talk, we will discuss a necessary and sufficient condition that an abelian representation of a certain finitely presentable group deforms to a nonabelian representation. Using the condition, we can show that the twisted Alexander invariant, which is defined as a rational function, of surface bundles over the circle becomes a polynomial for any nonabelian $SL(2;F)$-representation. As a corollary, we show that the twisted Lefschetz zeta function also becomes a polynomial.

A Kontsevich Integral for Fox Coloured Knots

Daniel Moskovich (RIMS Kyoto University)

Abstract

The Kontsevich integral of a knot may be obtained from the LMO invariant of its branched cyclic cover. For p-coloured knots, an analogous invariant may be obtained from the LMO invariant of its irregular branched dihedral cover. The 1-loop part gives a non-commutative analogue of the Alexander polynomial for knots equipped with a non-trivial Fox colouring.

Seifert matrices of Brunnian links and Programs

Maki Nagura (Yokohama National University)

Abstract

We show properties of a Seifert matrix of an n-component Brunnian link and prove some properties of the Alexander-Conway polynomials of Brunnian links. In particular, we give a sufficient condition for a matrix to be a Seifert matrix of a 2-component Brunnian link. Finally, we show programs for computing Seifert matrices of 2-component Brunnian links.

Construction of fibered knots with given Alexander polynomials via braidzel surfaces

Takuji Nakamura (Osaka Electro-Communication University)

Abstract

As a generalization of pretzel surfaces, L. Rudolph has introduced a notion of braidzel surfaces on his study of the quasipositivity for pretzel surfaces. It is well-known that for any reciprocal Laurent polynomial $f(t)$ with $f(1) \doteq 1$ there exists a knot whose Alexander polynomial is $f(t)$. In this talk, we construct a knot of unknotting number one via a braidzel surface with a given Alexander polynomial. We show that this knot is fibered with a braidzel surface as a fiber if a given Alexander polynomial is monic. We also construct infinitely many fibered knots with the same Alexander polynomial from our example.

Ordering knot groups

Dale Rolfsen (University of British Columbia)

Abstract

Composite double torus knots in 3-manifolds

Toshio Saito (Osaka University)

Abstract

Let M be a closed orientable 3-manifold. A knot K is called a *double torus knot in M* if K is isotopic into a genus two Heegaard surface of M. It is well-known that a $(1,1)$-knot is a tunnel number one knot and that a tunnel number one knot is a double torus knot. However, it is also known that the converse does not hold. In fact, it is shown that there is infinitely many tunnel number one knots which do not admit $(1,1)$-splittings. It is also proved that any tunnel number one knot in S^3 is prime. However, there is a composite double torus knot in S^3. Precisely, it is proved by Morimoto that every composite double torus knot in S^3 is obtained by the connected sum of two torus knots.

In this talk, we characterize the composite double torus knots in a closed, orientable, prime 3-manifold M.

Sheet numbers and cocycle invariants of surface-knots

Shin Satoh
(Chiba University)

Abstract

The sheet number of a surface-knot is the minimal number of broken sheets among all possible diagrams of the knot. It is a generalization of the crossing number of a classical knot. On the other hand, the cocycle invariant is a state-sum invariant of an oriented surface-knot defined by using a 3-cocycle of the quandle cohomology. We give a lower bound of the sheet number if the cocycle invariant of a surface-knot is not constant. As a corollary, we prove that the 2- and 3-twist-spun trefoils have the sheet number four and five, respectively.

Charts with at most one crossing

Akiko Shima (a joint work with Teruo Nagase) (Tokai University)

Abstract

We show the following:

Main Theorem. *Any chart with at most one crossing is a ribbon, i.e., any chart with at most one crossing is C-move equivalent to a chart without white vertices.*

Let Γ be a chart, (e_1, e_2, e_3) a triplet of edges of Γ. If there exists an embedding f of a disk D to the 2-sphere S^2 with $f(D) \cap \Gamma = e_1 \cup e_2 \cup e_3$, if both of $e_1 \cap e_2$ and $e_2 \cap e_3$ are one white vertex, and if e_1 is a terminal edge, then the triplet (e_1, e_2, e_3) is called a *consecutive* triplet. Here a *terminal edge* means an edge of Γ containing a white vertex and a black vertex. The following is the main idea in our talk:

Lemma. *Let* (e_1, e_2, e_3) *be a consecutive triplet in a k-minimal chart. Then the label of* e_1 *is different from the one of* e_3 (*we call it admissible*).

Here a *k-minimal chart* means as follows: For each chart Γ, let $w(\Gamma)$, $f(\Gamma)$, and $t(\Gamma)$ be the number of white vertices, the number of free edges, and the number of tiny convex lenses in Γ respectively. Let $C(\Gamma) = (w(\Gamma), -f(\Gamma), -t(\Gamma))$. The triplet $C(\Gamma)$ is called an *extened complexity* of the chart Γ. For each non-negative integer k, let

$C_k = \{\Gamma' \mid \text{ a chart } \Gamma' \text{ contains at most } k \text{ crossings}\}$.

A chart Γ in C_k is said to be *k-minimal* if its extended complexity is minimal among the charts in C_k which are C-move equivalent to the chart Γ with respect to the lexicographical order. A *tiny convex lens* means the closure of an open disk E such that ∂E consists of two edges e_1 and e_2 of Γ and any edge containing a white vertex in e_1 does not intersect the open disk E.

A formula for the colored Jones polynomial of 2-bridge knots

Toshie Takata (Niigata University)

Abstract

We derive a formula for the colored Jones polynomial of 2-bridge knots. For a twist knot, a more explicit formula is given and it leads to a relation between the degree of the colored Jones polynomial and the crossing number, which Le derived for a non-trivial alternating knot.

The almost alternating diagrams of the trivial knot.

Tatsuya Tsukamoto (Waseda Univeristy, JSPS)

Abstract

Answering to the question by C.Adams et.al. we show how to generate all the almost alternating diagrams of the trivial knot.

On the Casson-Walker invariant of branched covers of fibered knots

Yukihiro Tsutsumi (Sophia University)

Abstract

We show that for any integer $g > 1$ and a monic knot-Alexander polynomial $f(t)$ of degree g, there are fibered knots K_m ($m = 0, \pm 1, \pm 2, \dots$) such that $\Delta_{K_m}(t) = f(t)$ and $\lambda(\Sigma^2_{K_m}) - \lambda(\Sigma^2_{K_0}) = 4m$.

Boundary links are self delta-equivalent to trivial links

Tetsuo Shibuya (Osaka Institute of Technology)
Akira Yasuhara (Tokyo Gakugei University)

Abstract

Self Δ-equivalence is an equivalence relation for links, which is stronger than link-homotopy defined by J. W. Milnor. It is shown that any boundary link is link-homotopic to a trivial link by L. Cervantes and R. A. Fenn and by D. Dimovski independently. In this paper, we will show that any boundary link is self Δ-equivalent to a trivial link.

Surface diagrams and their complexes

Tsukasa Yashiro (University of Auckland)

Abstract

A surface-knot is an embedded oriented closed connected surface in 4–space. A surface diagram is a projected image in 3-space of a surface-knot. In this talk we show that a surface diagram is represented by a complex, which may consist of vertices, edges, rectangles and discs. We define local deformations of such a complex. We show that for two equivalent surface diagrams, there is a finite sequence of those local deformations between their complexes.