Research plan

Takanori Ayano

In [Shaska 2001], a canonical form of the defining equation of a hyperelliptic curve of genus 2 which admits a morphism of degree 3 to an elliptic curve is given. In [Shaska, Wijesiri, Wolf, Woodland 2008], a canonical form of the defining equation of a hyperelliptic curve of genus 2 which admits a morphism of degree 4 to an elliptic curve is given. Under these conditions, I will express the Abelian functions of genus 2 in terms of the Weierstrass elliptic functions. Let $f(x)$ be a polynomial of x. Let $\mathcal{X}$ be the hyperelliptic curve defined by $y^2 = f(x^2)$, $\mathcal{X}_1$ be the hyperelliptic curve defined by $y^2 = f(x)$, and $\mathcal{X}_2$ be the hyperelliptic curve defined by $y^2 = xf(x)$. It is known that the Jacobian variety of $\mathcal{X}$ is isogenous to the direct product of the Jacobian varieties of $\mathcal{X}_1$ and $\mathcal{X}_2$. I will express the Abelian function associated with $\mathcal{X}$ in terms of the Abelian functions associated with $\mathcal{X}_1$ and $\mathcal{X}_2$. It is well known that the Abelian functions satisfy the KdV–equations and the KP–equations. By using the formulae to express the Abelian functions in terms of the elliptic functions, it is possible to compute the values of the Abelian functions by using the mathematical software such as Mathematica.

Let $F(x)$ be a polynomial of x with degree $2g+2$, V be the hyperelliptic curve of genus g defined by $y^2 = F(x)$, and α be a complex number such that $F(\alpha) = 0$. For $1 \leq i \leq g$, we consider the holormorphic 1-forms $\omega_i = \frac{x^{i-1}}{y}dx$. Let $\omega = {}^t(\omega_1, \ldots, \omega_g)$. For $P_i \in V$, let

$$u = \sum_{i=1}^{g} \int_{(\alpha,0)}^{P_i} \omega.$$

I will express the coordinates of P_i in terms of u, which is the Jacobi inversion problem for the hyperelliptic curve defined by the polynomial with even degree. I will derive differential equations satisfied by the functions which give solutions to the inversion problem. This problem is solved in Whittaker and Watson, A Course of Modern Analysis for $g = 1$. I will extend this result to the case of $g \geq 2$. In mathematical physics, a hyperelliptic curve defined by a polynomial with even degree appears frequently. It is important to construct the theory of Abelian functions for the hyperelliptic curves defined by polynomials with even degree as well as the hyperelliptic curves defined by polynomials with odd degree.

For the hyperelliptic curves of genus $g = 2, 3$, I derived the partial differential equations integrable by the meromorphic functions that satisfy $2g$ periodicity conditions on the zero set of the sigma functions, which is a joint work with V. M. Buchstaber. These partial differential equations are two parametric deformations of the KdV–equations. The zero set of the sigma function associated with the hyperelliptic curve of genus g is equal to the image of $g-1$ points on the curve by the Abel–Jacobi map. For the hyperelliptic curves of genus g, I will derive partial differential equations integrable by the meromorphic functions that satisfy $2g$ periodicity conditions on the image of $k(< g)$ points on the curve by the Abel–Jacobi map. I think that these partial differential equations will be $2g-2k$ parametric deformations of the KdV–equations.