

Future Research

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As before, we will conduct research to develop existing theorems into theorems in CAT space. We investigate fixed point approximation theorems in Hilbert space, Banach space, and Hadamard space (complete $CAT(0)$ space), and consider what theorems can be developed in $CAT(\kappa)$ space. In many cases, the recurrence formula that approximates the fixed point can be used as is. On the other hand, we will need to devise reasonable assumptions for the mapping. It is well known that in Hilbert space, a projection or resolve to a nonempty closed convex set becomes a nonexpansive mapping. On the other hand, although counterparts can be defined in $CAT(1)$ space, they have poor properties and are not nonexpansive. When researching the development of existing theorems in $CAT(1)$ space, the basic approach is to first conduct research with the assumption that the mapping that approximates a fixed point is nonexpansive (although there are few applications). If the expected result can be proven, it can be generalized to the assumption of "strongly quasi-nonextended and Δ -demiclosed" (this assumption includes nonexpansive mappings and resolvents in $CAT(1)$ space are included). Recently, we have also been paying attention to properties such as "asymptotically nonexpansive" and " I -nonexpansive" for a given mapping I , and the direction of generalization after setting the assumption "non-extension" and obtaining results. We plan to conduct several such events. Specifically, the goals are as follows: In [1], when one convex function and one mapping are given in $ACT(1)$ space, a theorem is presented that simultaneously approximates the minimization point and fixed point. has been done. On the other hand, in [2] and [3], when two convex functions and maps are given in Hadamard space, the recurrence formula used in [1] is changed to the method of [2] and [3]. We plan to conduct research to develop the common fixed point approximation theorem, which is expected in nature, by multiplexing it using the following method. As a method, it seems that this can be achieved by carefully reading and applying the inequality evaluations performed in the preceding papers, as has been done up until now. In particular, the "parallelogram law" that holds in $CAT(1)$ space involves trigonometric functions and is difficult to handle. We will consider inequality evaluation that overcomes this obstacle. There is also a possibility that the research will be conducted in collaboration with Yasunori Kimura (Toho University) and Keisuke Shindo (National Institute of Technology, Hachinohe).

Reference

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[3] Khatoun, Sabiya; Cholamjiak, Watcharaporn; Uddin, Izhar, A modified proximal point algorithm involving nearly asymptotically quasi-nonexpansive mappings, *J. Inequal. Appl.*(2021), Paper No. 83, 20 pp.