

Future research plans

My aim is to develop the theory of stability of solitary waves in nonlinear dispersive equations by solving problems that have been brought to light by my previous researches. My research interests include the following three topics.

(1) *Studies on two-parameter families of solitary waves.* Derivative nonlinear Schrödinger equations and coupled systems of nonlinear Schrödinger equations have families of solitary waves that essentially depend on two parameters: frequency and velocity. Previous studies have mainly dealt with one-parameter families. On the other hand, for two-parameter families, there are many unresolved issues, including the basic properties (symmetry, uniqueness, and nondegeneracy) of the solutions for nonlinear elliptic equations. Moreover, methods for applying abstract theory have not been established. This project aims to develop methods for analyzing these properties and applying abstract theory to two-parameter families of solitary waves.

Recently, scattering and blowup of radially symmetric solutions with smaller action than standing waves have been studied for non Galilean invariant equations such as nonlinear Schrödinger systems and fractional nonlinear Schrödinger equations. On the other hand, the dynamics of solutions without radial symmetry are still unknown. I expect that traveling waves, rather than standing waves, will play an important role in the dynamics of nonradial solutions, and that their properties should be investigated in detail. In future research, I will investigate the relationship between traveling waves and scattering and blowup solutions without radial symmetry, and aim to investigate the global dynamics of solutions.

(2) *Study of algebraic solitary waves.* Usually solitary waves decay exponentially, but in critical situations, algebraic solitary waves with polynomial decay may appear. In paper [6], a partial result on instability was obtained, but it is not optimal. Also, stable algebraic solitary waves have not been found. Therefore, we first develop a theory of algebraic solitary waves of the double power nonlinear Schrödinger equation from the viewpoint of nonlinear elliptic equation theory and operator theory. Moreover, we aim to construct a new theory of stability and instability of the solitary waves. In addition, we will also study the stability and instability of algebraic solitary waves for other equations.

(3) *Asymptotic stability and strong instability of solitary waves.* When the equations are not scale or Galilean invariant, the stability of solitary waves essentially depends on parameters such as frequency and traveling speed, and asymptotically stable and strongly unstable waves appear. However, asymptotic stability and strong instability remain unresolved for many equations, and questions such as how they vary with frequency and travel speed are unknown. Therefore, we consider these problems for various equations, including the nonlinear Schrödinger equation with potential or point interaction, and aim to clarify the universal conditions for asymptotic stability and strong instability to occur.