

RESEARCH ACHIEVEMENTS

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Since I started working on my thesis at the Johns Hopkins University, I have been working on automorphic representations and automorphic L -functions. Below is a brief description of my research achievements up to now.

On integral representations of automorphic L -functions

In my thesis (Amer. J. Math. 1993) written under Joseph Shalika, an integral representation for the automorphic L -function of $\mathrm{SO}(4) \times \mathrm{SO}(2)$ is obtained as a Bessel period of Eisenstein series on $\mathrm{SO}(5, 2)$. This precedes the general result obtained by Ginzburg, Piatetski-Shapiro and Rallis concerning the integral representation for $\mathrm{SO}(V) \times \mathrm{GL}(n)$.

In a later paper (Crelle 1993), an integral representation of the degree eight L -function for $\mathrm{GSp}(4) \times \mathrm{GL}(2)$ is obtained utilizing the Klingen type Eisenstein series on $\mathrm{GU}(2, 2)$. As an application, the algebraicity of the rightmost critical value is proved. Later by Pitale, Saha and Schmidt, this integral representation is used to prove the transfer from $\mathrm{GSp}(4)$ to $\mathrm{GL}(4)$ by the converse theorem method.

In a joint work with Bump and Ginzburg (Crelle 1995), new integral representations of automorphic L -functions which are based on non-unique local models are discovered. This subject is revitalized recently by Pollack, Ginzburg and others.

On special values of automorphic L -functions

In a joint work (Amer. J. Math. 2014, 2016) with Kazuki Morimoto at Kobe University, utilizing the integral representation by Ginzburg, Piatetski-Shapiro and Rallis mentioned above, the algebraicity of the critical values of the automorphic L -functions for $\mathrm{SO}(V) \times \mathrm{GL}(2)$, where V is a definite quadratic space, is proved. This result includes the unbalanced case of the Rankin triple L -function.

In the 1980's, Böcherer proclaimed a very interesting conjecture concerning the relationship between a finite sum over the ideal class group of an imaginary quadratic field of Fourier coefficients of a degree two Siegel cusp form which is a Hecke eigenform and a central value of the spinor L -function twisted by the corresponding quadratic character. Since the late 1990's, I pursued an approach to Böcherer's conjecture based on the relative trace formula method, with Shalika and later also with Martin. We succeeded in proving the fundamental lemma and its extension to the entire Hecke algebra (Memoirs of AMS 2003, 2013, Amer. J. Math. 2011) but later we got stuck. As a joint work with Morimoto, we reconsidered the initial approach in the early 1990's based on theta correspondence (Crelle 1995). Thanks to the remarkable progress in theta correspondence and the breaking ground formulation of the explicit formula for the central special values by Ichino and Ikeda, we succeeded in proving Böcherer's conjecture (JEMS 2021). The 2022 MSJ Algebra Prize was awarded for the work.