Future works

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In the papers [10, 8, 4, 1], the scalar-potential representation formulae for curl-free or solenoidal fields play an essential role, which enable us to solve the minimization problem concerning the improvement of the best constants in Hardy and Rellich inequalities. The tools seem to be applicable to analyze many other functional inequalities of Hardy type. In particular, for further development, I intend to work with the following topics:

- The remainder terms of Hardy-type inequalities for constrained vector fields includes information about estimating the differences between the two integrals in each of the inequalities. In some sense, I am planning to obtain a sharp form of the remainder terms.

- Rellich-Hardy inequality, which can be considered as an intermediate between Hardy and Rellich inequalities, was shown by Tertikas-Zographopoulos, together with its weighted version. Since the expression for the best constants for curl-free or solenoidal fields was solved in recent papers [12] and [2] in the Rellich-Hardy inequalities including any radial power weights, I'm interested in solving for more general weight form.

- My research so far have concerned only L^2 type of Hardy-Leray inequality, but the general L^p type for $p \neq 2$ also has a curl-free or solenoidal improvement. An exact computation of the best constant for $p \neq 2$ is yet unknown and remains very difficult, since the method of spherical harmonics expansion is not applicable at all; I am seeking for the approach by constructing a rearrangement technique subject to not radial but axial symmetrization.

- My research so far have treated Hardy-Leray inequality on the whole Euclidean space, while the inequality with trace remainder term on the half space is known as Kato's inequality. I am interested in the problem whether this inequality for vector fields is more sharpened by assuming the curl-free or solenoidal condition.

- The best constant in Heisenberg's uncertainty principle inequality was computed in the paper [3] for solenoidal fields. This inequality together with Hardy's inequality are included in the class of Caffarelli-Kohn-Nirenberg's inequalities. I aim to investigate whether the best constants are all computable in such class for constrained vector fields.

- When the definition domain of the test fields is curved (or more precisely it has a constant curvature), the best constant is expected to be changed in functional inequalities for solenoidal or curl-free fields; I am interested in what value does it takes and would like to solve its explicit expression.

Since the sharpness of the inequalities is closely related to their Euler-Lagrange equations, I am also seeking for the possibility of an application of the present research to the theory of partial differential equations such as Navier-Stokes equations; by way of the solenoidal improvement, it can be expected to give a new perspective around the Leary's existence theory for weak solutions.