• S. Hamanaka, Type of finite time singularities of the Ricci flow with bounded scalar curvature, arXiv:2105.08250 (2021).(Not peer-reviewed)

In this paper, I showed that the shape of finite-time singularities of a Ricci flow on a closed manifold with bounded scalar curvature are restricted in some sense.

• S. Hamanaka, Limit theorems for the total scalar curvature (old version: C^0 , C^1 -limit theorems for total scalar curvatures), arXiv:2208.01865 (2022).(Submitted)

In this paper, I particulary proved the following: Let \mathcal{M} be the space of all Riemannian metrics on a closed *n*-manifold M ($n \geq 3$). Then, for any nonnegative continuous function σ and constant κ , the space

$$\left\{g \in \mathcal{M} \mid \int_M R(g) \, d\mathrm{vol}_g \ge \kappa, \ R(g) \ge \sigma \right\}$$

is closed in \mathcal{M} with respect to $W^{1,p}$ (p > n)-topology.

In the same paper, I also proved that the lower bound of the weighted total scalar curvature

$$\int_M R(g) e^{-f} \, d\mathrm{vol}_g$$

is preserved under certain convergences of metrics and weight functions. Here, f is some weight function on the manifold M. As a corollary of this weighted version, we can also obtain that the scalar curvature lower bound in the distributional sense (which was defined by Lee–LeFloch) is preserved under $W^{1,p}$ ($p > n^2/2$)-convergence of metrics provided that the limiting metric is C^2 .

• S. Hamanaka, Upper bound preservation of the total scalar curvature in a conformal class, in preparation. (Not peer-reviewed)

In this paper, I particulary proved the following: Let $g_0 \in \mathcal{M}$. Then, for any continuous function σ and constant κ , when the conformal class $[g_0]$ of g_0 is Yamabe positive or nonpositive respectively,

$$\left\{g \in [g_0] \mid \int_M R(g) \, d\mathrm{vol}_g \le \kappa, \ R(g) \ge \sigma\right\}, \ \left\{g \in [g_0] \mid \int_M R(g) \, d\mathrm{vol}_g \le \kappa\right\}$$

are closed in \mathcal{M} with respect to C^0 -topology respectively.