Research and Achievements

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1 Abstract

My study is to investigate the properties of stochastic processes on Riemannian manifolds. The following figures show simulated sample paths of the Brownian motion on Euclidean space and the Brownian motion on the upper half-plane equipped with the Poincaŕe metric.

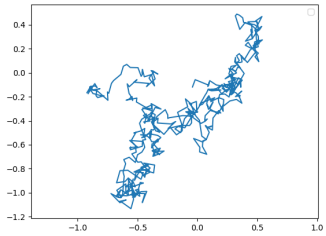

図 1　Brownian motion on $\boldsymbol{R}^2$

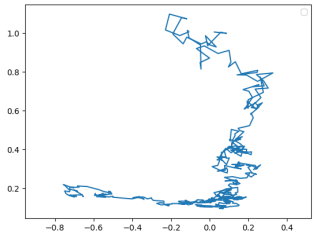

図 2　Brownian motion on $\boldsymbol{H}^2$

From these simulations, we can see that the Brownian motion on the upper half-plane is more strongly affected by the force diverging to the infinity than the Brownian motion on the Euclidean space.

My study is to reveal how the curvature on a Riemannian manifold affects the path-properties of the stochastic process.

2 My reserches

The Lévy process is an important class of stochastic processes on the Euclidean space. Applebaum extended the Elles-Elworthy-Malliavin construction to construct the Lévy process on Riemannian manifolds. From so far, I have been focusing on such processes and studied the followings:

(1) Conditions for the law of the Lévy process on Riemannian manifolds to have density function.

(2) The long time behavior of the Lévy process on Riemannian manifolds.

(1): In a joint work with Professor Atsushi Takeuchi of Tokyo Woman's Christian University, Kai-Takeuchi (2021) solved this problem by deriving a Bismut-type integration by parts formula. By this work, we can guarantee the existence of the heat kernel for the Lévy process.

(2): In Kai (2024), we showed that the Lévy process on Cartan-Hadamard manifolds is irreducible and that we can evaluate the radial part of the process if the sectional curvature is pinched by negative constants. From these results, I revealed that the Lévy process is transient and conservative.