Future Research Plan

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In the beginning of my research career, I studied knot theory and felt some algebraic invariants of knots attractive, especially Alexander polynomials. Thus the advisor (Shinji Fukuhara) and I were happy to define a new invariant, a set of matrices, for 3-manifolds through Heegaard splittings by using the computation process for Alexander polynomials; we named the new invariant “Extended Alexander matrices” (1985). Later, Shinji Fukuhara published its applications for Lens spaces; there exist two Lens spaces that cannot be distinguished by their fundamental groups, but can be distinguished by the Extended Alexander matrices. I wonder now if we can find further information or applications on Extended Alexander metrices; if there are more 3-manifolds, other than Lens spaces, on which Extended Alexander matrices play a meaning role. Also, I am interested in seeking a relation of the new invariant to already established other algebraic invariants for knots or 3-manifolds. At that time, Akio Kawauchi predicted such a possibility but I did not have a chance to pursue that direction. Since then there have been so many new developments in topology, I want to give me a chance to learn those new developments and to investigate what the Extended Alexander matrices mean through the different tools and languages after four decade.

After finishing graduate school in Japan, I started studying topological graph theory and continued learning graph theory in the rest of my working career. I have realized that the concept of topology has been foreign to the most graph theorists; I know this from bitter experiences. Graph Minors are popular in topological graph theory such as Robertson-Seymour graph minor publications; however, Graph Immersions are studied by a few groups of researchers. Since I am one of them, I feel responsible to explain the concept of graph immersion to more people. I used the concept as a containment relation between two graphs and proved Splitter theorems for 3-regular graphs and 4-regular graphs with some restrictions on girth and edge connectivity. I have submitted a paper titled “Splitter theorems for 4-regular planar graphs,” but it was returned because the length of the paper did not match the journal. This coming year I want to review and shorten my writing so that I can submit to a journal.

Currently, I am working with colleagues on Kauffman's bracket polynomials, that is, three variable polynomials defined for knot diagrams and how we can make them be a knot invariant. To make them be a knot invariant, we have to consider Reidemeister moves since if two knot diagrams are equivalent through three Reidemeister moves (R-1, R-2 and R-3), then the two knots are equivalent. This topological information can be transformed to algebraic language. Each Kauffman polynomial is an element of three variable integral coefficient polynomials denoted Z[A,B,d] and we consider a quotient map divided by the ideal generated by the Reidemeister moves. In fact, we are investigating two ideals I_1 and I_2 where I_1 is generated only by R-2 and I_2 is generated by all three R-moves. When we learned about the Groebner basis in field coefficient polynomials, we experimented to find the Groebner basis by assuming the I_1q and I_2q are ideals in Q[A,B,d]. As a result, we proved that the I_2q is a sub-ideal of I_1q by using Groebner basis. The I_1q has the reduced Groebner basis (unique) consists of three elements and the I_2q has the basis consists of four elements. We do not know yet how this extension from Z[A,B,d] to Q[A,B,d] affects. We hope the affect is not too bad since the basis of I_2q uses all integer coefficient except the fraction ½.