Summary of Research Results
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January 28, 2024

At first, I expanded from the dissertation in graph theory with key words, such as splitter theorems [List 9,10,14], generating theorems, graph minors, graph immersion, *k*-regular graphs for $k = 3,4,5$, together with girth and connectivity restrictions etc. Proving a splitter theorem for a given class of graphs under a certain containment ($\leq$) means that we provide a set S of graph operations, which change only local structure at a time, for any pair of graphs (H, G) such that whenever $H \leq G$ there exists a series of graph operations in S starting from H building to G, or equivalently starting from G reducing to H where for every consecutive pair the containment relation is maintained. As a result, every graph between G and H contains H, which is unchanged during the reduction process. If we have a splitter theorem for a class C of graphs with a containment relation, then we can prove a generating theorem as a corollary by setting H as the empty graph. In other words, we can set up the set of minimal graphs in C by collecting the graphs just right before the empty graphs. My original motivation to these studies was to prove a generating theorem for 5-regular simple planar graphs [List 13], but we may need a new approach to prove a clean generating theorem for this class.

Next, I focused on directed graphs and book embeddings since the literature does not produce much results. To tackle together both of them, I defined a new rule for directed graphs (digraph for short) how to be embedded into each page of the book space; each page must have a unique direction for all arcs embedded in that page: to be Up or Down when we identify the spine of the book with z-axis in x-y-z-axis space that contains the book space as a Euclidean subspace. According to the rule, we can define an oriented book thickness of a digraph D, denoted obt(D), as the smallest number of pages for D to be embeddable without violating the rule. To characterize digraphs according to $obt(D) = k$, we define a digraph *D* to be *k-critical* if and only if $obt(D) = k$ but every proper subgraph D' of D has a property $obt(D') < k$. We provide a complete list of *k*-critical digraphs for $k = 0$ and $k = 1$, and for the class $\mathfrak{U}$ of unicycle digraphs that contains at most one directed cycle, we determine the list of 2-critical digraphs of $\mathfrak{U}$ [List 7].

From the oriented book embedding, there are at least two directions of developments: one is the concept of a quasi-surface which is the compactification of the book space and the other is a study of digraphs together with the line graph operator. More precisely taking the compactification of the spine of the book, the z-axis, results in a circle called the Event Horizon. Then one page book with a circular spine is homeomorphic to the closed 2-disc; the two-page book with the circular spine is homeomorphic to S^2 by seeing the surface of the earth with the equator as the Event Horizon. The k-page book with the circular spine will be denoted $\mathbb{T}_k$. We proved the Euler number of $\mathbb{T}_k$ can be defined and it is k [List 6].

Also, applying the line graph operator on digraphs repeatedly results in three categories: dissipated, expanded, and eventually periodic. We focus on the third status and want to describe more details than previously existed papers. To do this, we re-formalized the concepts more algebraically and precisely by defining more terminologies and detailed formulas. This requires more careful treatment since the line graph operator is not a homomorphism [List 1].