Future research plans   Kotaro Kawai

I want to construct an invariant by G_2-dDT connections. I expect to obtain more interesting results in the case of G_2-dDT connections. The reason is: (i) (More precisely,) G_2-dDT connections correspond to "graphical" calibrated submanifolds, and the singular set seems to be more tractable. (ii) From [15, 16], the moduli space of G_2-dDT connections seems to have better properties than calibrated submanifolds and G_2-instantons. (iii) Significant research has been done on dHYM connections, with deeper results than calibrated submanifolds.

[Plan 1] Compactness theorem for minimal connections

To construct invariants, it is necessary to study the compactification of the moduli space and investigate its properties in detail. To do this, we first need to consider the compactness theorem of minimal connections.

The analogy of "(i) Price's monotonicity theorem" was shown in paper [22], but the result might be made a little stronger. In fact, we were able to show a stronger statement for G_2-dDT connections. I would like to consider how strong it can be made by studying examples. It also seems important to find a connection between the monotonicity theorem and scaling (in some sense) of the "volume" V.

For the analogy to "(ii) Uhlenbeck-Nakajima ε regularity theorem", it is necessary to show that the "energy density" (integrand) v of V satisfies the "Bochner-type inequality". (That is, v should be a subsolution of an elliptic operator of divergence form.) Since v is much more complicated than the energy density of the Yang–Mills functional, it will be more difficult to prove such an inequality. However, I showed in [22] a similar formula to the Weitzenböck formula, which was important in the proof of the Yang–Mills case, and found that terms with the highest derivative would be handled well. The low-order derivative terms still seems to be complicated, but I expect that they can be estimated well with some more technical ingenuity.

[Plan 2] (Morse-)Floer homology using G_2-dDT connections

From "My achievements (Ⅲ)", there is an observation for $G_2, \mathrm{Spin}(7)$-dDT connections analogous to that of instanton Floer homology (IFH) for 3-manifolds. As a further development of [Plan 1], we might be able to construct a Floer homology using G_2-dDT connections.

As analogies to IFH, I will first consider whether (a) the deformation complex of Spin(7)-dDT connections on the cylinder is Fredholm and its (relative Morse) index is given "nicely", and (b) the moduli space is smooth by the "holonomy perturbation".

The same argument as in the case of IFH seems to work. However, the observation in [21] only holds under "certain conditions". Considerable technical difficulties are expected due to this. However, since the difficulty is mainly due to the complexity of $G_2, \mathrm{Spin}(7)$ geometry, I expect that it will be possible to be solved with the method developed in [13–16] and [Plan 1]. Then, referring to other Floer homology theories, I would like to consider the following. (1) Is it possible to compactify the moduli space of Spin(7)-dDT connections on the cylinder? (2) If the Floer homology can be constructed, to what extent does it depend on the geometric structures and perturbation? (3) Can we define a higher-order product structure such as the A_∞ structure for this Floer homology?

[Plan 3] "Mirror" of mean curvature flow (MCF)

We showed the short-time existence and uniqueness of the "mirror" of mean curvature flow in [16]. Many of the results for the standard MCF for submanifolds are expected to hold for "mirror" MCF as well.

From [16], $G_2, \mathrm{Spin}(7)$-dDT connections minimize the "volume". So I would like to study the stabilities of the $G_2, \mathrm{Spin}(7)$-dDT connections. (That is, I will check if the mirror MCF converges to a $G_2, \mathrm{Spin}(7)$-dDT connection if the flow starts close enough to them.) To prove this, I am considering the analogue of the method of H. Li for Lagrangian MCFs. That is, under the assumption that the first eigenvalue of the linearized equation is sufficiently large, I will show that L^2 norm of the mirror mean curvature decays exponentially, etc. This statement is expected to hold from the mirror correspondence, but I will study with reference to other flows whose stability is known (such as Laplacian flow on G_2-manifolds) as appropriate.