Masahide Manabe

Research proposal

2D gravity and topological recursions. The CEO (Chekhov-Eynard-Orantin) topological recursion can be universally applied to a broad class of theories related to 2D gravity. Inspired by this recursion, in 2017, for moduli spaces of bordered Riemann surfaces, the ABO (Andersen-Borot-Orantin) topological recursion, which is a generalization of the Mirzakhani's recursion for the Weil-Petersson volumes, and the quantum Airy structures (by Kontsevich-Soibelman and Andersen-Borot-Chekhov-Orantin), which give a generalization of the Virasoro constraints, are proposed. In [22] of "List of Publications", by using these three approaches (CEO topological recursion, ABO topological recursion, quantum Airy structures), we studied the 2D $(2,p)$ minimal gravity with an odd integer p and its Masur-Veech type twist. Here, the volumes for the $(2,p)$ minimal gravity are considered as a generalization of the Weil-Petersson volumes obtained for $p \to \infty$. It is interesting to study other models related to 2D gravity by applying these approaches. Recently, with Hiroyuki Fuji and Yoshiyuki Watabiki, we are discussing how the CEO topological recursion is obtained from the "Hamiltonian formalism" for the $(2,p)$ minimal gravity. The CEO topological recursion is considered to provide a wider framework than the "Hamiltonian formalism", and it is interesting to clarify what class of theories have "Hamiltonian formalism" for the topological recursion.

3D supersymmetric gauge theory and knot theory. We have constructed an abelian gauge theory in [20] of List of Publications, which we called "knot-gauge theories", whose K-theoretic vortex partition functions give the colored Jones polynomials of knots in S^3. For the construction, we utilized an exact formula of the A-twisted partition function (twisted index) of 3D $\mathcal{N}=2$ gauge theory on $S^2 \times S^1$ obtained by Benini-Zaffaroni in 2015, and the factorization of A-twisted partition functions into K-theoretic vortex partitions. About the knot-gauge theory, there are some problems to unveil as follows. Firstly, the knot-gauge theory provides a relation "colored Jones polynomial = K-theoretic vortex partition", and the right hand side is expected to be obtained as a generating function of Euler characteristics for moduli spaces of vortices. So, it is interesting to construct vortex moduli spaces and provide a geometric interpretation of the colored Jones polynomials. Secondly, the knot-gauge theory is labeled by tangle diagrams of knots and has R-matrix-based building blocks, and as a result, it is non-trivially transformed under the Reidemeister moves. It is important to understand this transformation in the knot-gauge theory. Thirdly, it would be interesting to introduce the parameter t of homological grading, by Dunfield-Gukov-Rasmussen, that categorifies the colored Jones polynomial, to the R-matrix. It is not known, for general knots, how the parameter t is introduced in the R-matrix formalism, and this is a mathematically challenging problem. Our construction of the knot-gauge theory is based on the computation of the colored Jones polynomial by the R-matrix, and this is also related to giving an interpretation of the parameter t in the knot-gauge theory.

Non-perturbative topological strings and refined topological strings. The topological string theory is perturbatively well-defined, and it is important to understand its non-perturbative nature. For this research direction, in 2008, Eynard and Marino proposed a method to provide non-perturbative corrections to the perturbative free energies by the CEO topological recursion. On the other hand, it is also known that, for the refined (one-parameter deformed) topological string theory on local toric CY3s, the so called NS (Nekrasov-Shatashvili) limit provides a non-perturbative correction to (unrefined) topological strings. It is interesting to clarify the relation between the proposal by Eynard-Marino and non-perturbative corrections by the NS limit. Here, the refined topological string theory is also known to be related with the categorification of the colored Jones polynomial in knot theory, and it is interesting to study relations with the non-perturbative nature in the context of "geometric topology", "algebraic geometry", and "enumerative geometry". Furthermore, a brane partition function in the topological string theory gives a quantization of spectral curves in target spaces, and the brane partition function in the refined topological string theory is considered to give a "double quantization" of spectral curves. I also want to understand relations between the double quantization and non-perturbative corrections.