(2) Study proposal Makiko Mase

(i) Aim of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces, which we simply call a $K3$ surface . The study of geometry of $K3$ surfaces is deeply related to classical areas such as singularity theory and algebraic curve theory, and modern area of mathematical physics. It is necessary to study the period mappings and the Picard lattices for our purpose, particularly related to Picard lattices, it is unavoidable to understand the behaviour of curves on the surfaces. Related to analytic geometry and mathematical physics, it is a long-lived problem to characterize $K3$ surfaces from the view point of the moduli spaces of maps from a $K3$ surface to a Lie algebra. We consider the following problems:

Problems

1. The relation between algebro-topological properties of singularities and $K3$ surfaces.
2. The $K3$ surfaces of Picard numbers 0 and 21.
3. Symplectic automorphisms of $K3$ surfaces.
4. Weierstrass semigroups of pointed curves and $K3$ surfaces.

(ii) Study methods

Problem 1 Following Ebeling's theorem, we are interested in studying a relation between coupling duality and a duality of reduced zeta functions of isolated hypersurface singularities when coupling is non-primitive. Concerning simple $K3$ singularities and weighted $K3$ surfaces, we'd like to study if there is a relation between the Milnor lattice with Seifert form and the Picard lattice with intersection form.

Problem 2 We know that complex algebraic $K3$ surfaces have Picard number between 1 and 20. $K3$ surfaces defined over some fields of characteristic 2 may have Picard number 22. If a $K3$ surface has Picard number 0 (resp. 21), then, it means that the surface has no projective model (resp. the rank of the transcendental lattice becomes 1). Note that any complex algebraic $K3$ surface admits the transcendental lattice of signature $(2, 20 - \rho)$. We'd like to understand geometric structure of $K3$ surfaces with Picard number 0 and 21.

Problem 3 If a $K3$ surface X admits a finite symplectic automorphism group G, then, the quotient space X/G is birational to a $K3$ surface. Let L be the lattice generated by all classes of (-2)-curves in the exceptional divisor of the minimal resolution $Y \to X/G$. Note that L is not necessarily a primitive lattice in $H^2(Y, \mathbb{Z})$. Our question is to determine whether or not there exist a primitive closure $\tilde{L}$ of L.

Problem 4 (A joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology) We are interested in studying which Weierstrass semigroups for pointed curves are on/off a $K3$ surface, and we'd also like to give a characterisation for Weierstrass semigroups attained by an algebraic curve on a $K3$ surface that is the triple covering of a rational surface.

(iii) Aspects

In the consequence, we will be able to understand a relation between two profiles : algebraic geometry (Picard lattices) and topology (Milnor lattice) for $K3$ surfaces. We expect to study some differential-geometric objects in order to understand $K3$ surfaces of Picard number 0 or 21. By using the lattice theory and the algebraic geometry for subvarieties, the applicant expects that the above proposals help revealing the secret of those relations.