(2) Abstract of results Makiko Mase

Part I. Dualities of families of $K3$ surfaces associated to coupling duality

The aim of the study is to understand the notion of coupling duality introduced by Ebeling in terms of other relations concerning $K3$ surfaces. The $K3$ *weight systems* are well-posed weight systems $(a_0, a_1, a_2; d)$ of positive integers such that general quasihomogeneous polynomials of degree d with weights (a_0, a_1, a_2) define simple $K3$ singularities. According to Yonemura, there exist 95 $K3$ weight systems up to isomorphism. The *(strongly) coupling duality* is defined between well-posed weight systems of positive integers.

We consider strongly coupling pairs among $K3$ weight systems. Our problems are as follows.

(1) Does the strongly coupling extend to polytope duality between families of $K3$ surfaces ?

(2) If a strongly coupling pair extends to polytope duality, then, does this pair extend to lattice duality of families of $K3$ surfaces ?

For the both problems, we give partially affirmative answers. In the first part, we can explicitely construct appropriate pairs of reflexive polytopes that give polytope duality, and in the second part, we can compute the Picard lattices of the families that give lattice duality for families.

Therefore, we can conclude that strong coupling duality has interpretations into the followings.

(i) an analogy of classical mirror symmetry, which is the symmetry of Hodge diamond between two Calabi-Yau varieties,

(ii) “Dolgachev-Nikulin mirror” for lattice-polarized $K3$ surfaces.

Part II. Weierstrass semigroup of a pointed curve and $K3$ surfaces (joint work with Professor Jiryo Komeda)

The aim of the study is to characterize algebraic curves on/off a $K3$ surface in terms of Wierstrass semigroups.

In the published paper, we prove the following two theorems:

(A) Let C be a hyperelliptic curve of genus g, and $\tilde{C}$ be its double covering curve of genus at least $g^2 + 4g + 6$. Then, the curve $\tilde{C}$ does not lie on any $K3$ surface. (This is an extension of Reid's theorem.)

(B) For a numerical semigroup H, define a map d_2 by $d_2(H) := \{h/2 \mid h \in H : \text{ even}\}$. We construct two infinite sequences $\{S_l\}$ and $\{H_l\}$ of Weierstrass semigroups by using the map d_2. Here, the series $\{S_l\}$ is symmetric, while $\{H_l\}$ is non-symmetric. Any semigroups in these sequences are attained by algebraic curves that are not lying on any $K3$ surface.

The theorems are proved by mainly applying Reid's criterion, and important results on Weierstrass semigroups of “double covering type”.