Research Plan

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Characterization of Gödel Universe as an Einstein-Dirac-Maxwell System

Background and Objectives: Specific spinors, such as Killing spinors, can define Riemannian structures and uniquely determine spacetime when forming solutions to the Einstein system under certain conditions. The applicant previously demonstrated that the G"odel universe can emerge from an Einstein-Maxwell-Scalar system in 4D spacetime, albeit with somewhat artificial scalar field assumptions. The aim of this research is to characterize the Gödel universe in 4-dimensional stationary spacetime as an Einstein system formed by specific spinors and electromagnetic fields with more natural assumptions.

Significance: The previous work modeled the 4D G"odel universe with an Einstein-Maxwell-Scalar system, linking it to a 3D Lorentz Sasaki manifold and $\mathbb{R}$. This approach, novel for incorporating 3D Lorentz spin geometry results, anticipates advancing Dirac spinor handling in 4D stationary spacetimes.

Methods: First, consider the 4-dimensional Gödel universe as a direct product of a 3-dimensional Lorentz Sasaki manifold and $\mathbb{R}$, and realize the 4-dimensional Gödel universe as a solution to the Einstein-Dirac system using spinors on the 3-dimensional Lorentz Sasaki manifold. This has already been partially completed. Next, while observing the previous results, consider what conditions need to be imposed on the 4-dimensional stationary spacetime to realize the Gödel universe and prove it under appropriate conditions.

Annual Plan: From April to June, examine as many geometric structure candidates as possible that realize the exact solutions mentioned earlier and investigate their specific examples to seek explicit representations of exact solutions. Then, from July to September, abstract the geometric properties and consider them in a setting with as much generality as possible. If desirable results are not obtained in the second phase of generalization and characterization, the results of the first phase will be published. Even this is considered significant as a study of exact solutions.

Study of Relativistic Rayleigh-Benard Convection

Background and Objectives: Rayleigh-Bénard convection, seen in phenomena like swirling miso soup, typically occurs in viscous fluids with a temperature gradient and is studied in non-relativistic contexts. This research aims to extend the study to relativistic viscous fluids in curved spacetimes, such as Schwarzschild spacetime, to understand their behavior.

Significance: Investigating the convection of relativistic viscous fluids with temperature in a gravitational field could help in qualitatively understanding, for example, the convection of air around the Earth.

Methods: Referring to the formulation of non-relativistic Rayleigh-Bénard convection, use ① the relativistic Fourier's law for heat flow and temperature, ② the relationship between heat flow and entropy according to the first law of thermodynamics for relativistic viscous fluids, to perform relativistic formulation. Then, analyze the behavior of relativistic Rayleigh-Bénard convection in Schwartzschild spacetime both analytically and numerically.

Annual Plan: Research the relativistic formulation of Rayleigh-Bénard convection from April to August. Conduct analyses in Schwartzschild spacetime