Research Results

Satsuki Matsuno

The applicant's research theme is the application of contact geometry to general relativity, and the following will describe the research conducted in 2023 on ① the study of the Einstein-Dirac-Maxwell system in static Sasakian spacetime, and ② the geometric properties of Sasakian-quasi-Killing spinors on 3-dimensional pseudo-Riemannian manifolds.

① **Study of the Einstein-Dirac-Maxwell system in static Sasakian spacetime**: In Sasakian manifolds, contact magnetic fields arise from currents along the Reeb vector field, described by Maxwell's equations. An exact solution combining these magnetic fields with charged dust fluid in 4D static Sasakian spacetime has been developed. These studies, based on macroscopic charged fluids, suggest the possibility of exact Einstein-Dirac-Maxwell system solutions using spinor fields with an $U(1)$ gauge field, more fundamental matters, for similar scenarios. Constructing solutions for the complex EDM system, especially using Sasakian-quasi-Killing spinor fields unique to Sasakian manifolds, marks a novel approach. This work has been documented and submitted for publication[*1].

② **Study of the geometric properties of Sasakian-quasi-Killing spinors on 3-dimensional pseudo-Riemannian manifolds**: Sasakian-quasi-Killing (SqK) spinors were defined by Friedrich et al. as a generalization of Killing spinors on Sasakian manifolds. SqK spinors on 3-dimensional Sasakian manifolds have been classified in particular. However, their geometric and physical properties have not been investigated. The aim of this research is to clarify the properties of SqK spinors on 3-dimensional pseudo-Riemannian Sasakian manifolds. Our research revealed the following:

(i) The Dirac current formed by special SqK spinors becomes the Reeb vector field

It is interesting that the Reeb vector field, important in contact structures, can be described by SqK spinors.

(ii) The motion of charged particles in a contact magnetic field can be described by the Dirac current of SqK spinors

Such motions of charged particles have been extensively studied, but a new perspective was obtained.

(iii) Almost all SqK spinors become solutions to the Einstein-Dirac system with a cosmological term, and special SqK spinors and contact electromagnetic fields constitute solutions to the Einstein-Dirac-Maxwell system

These systems are complex nonlinear systems, and being able to construct exact solutions could signify the physical relevance of SqK spinors.

(iv) Explicit representation of SqK spinors in a certain frame

Though their existence was shown, this is the first time an explicit representation has been obtained.

This research has been compiled into a paper[*2] and is currently under submission to a professional journal.

[*1] arXiv:2402.00420v1.

[*2] arXiv:2308.10432v2