Summary of past research

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1 Past reserach - 1

From the beginning, my research had been motivated by the most important problems in the homotopy theory at the time. For example, in

- Norihiko Minami, The Kervaire invariant one element and the double transfer」Topology, vol. 34 (1995) p.481-488.
- Norihiko Minami, The Adams spectral sequence and the triple transfer, American Journal of Mathematics, 117, no.4, 965—985.
- Norihiko Minami, The iterated transfer analogue of the new doomsday conjecture, Trans. Amer. Math. Soc. 351 (1999), no.6, 2325-2351.,

I had formulated "the new doomsday conjecture,"some finiteness conjecture on the stable homotopy groups of the sphere, generalizing the later proved Hopkins-Hill-Ravenel Kervaire invariant one element finiteness theorem, and presented some supporting evidence. However, I had come to the conclusion that the new doomsday conjecture is well beyond the scope of the traditional homotopy theoretical methods. Speaking of such a truly global homotopy theoretical problem, I had also worked on the Hopkins' chromatic splitting conjecture, and obtained some result:

- Norihiko Minami, On the chromatic tower, American Journal of Mathematics, 125 (2003), no. 3, 449-473,

However, once again, I had come to the conclusion that Hopkins' chromatic splitting conjecture is well beyond the scope of the traditional homotopy theoretical methods.

2 Past reserach - 2

As explained above, I had been convinced of the limitations of the traditional homotopy theoretical methods, I began to fool around other areas of mathematics in search of any possible clue.

One of them is the theory of $mathbbF_1$-schemes, which was developed by Connes, Consani, Koyama, Kurokawa in order to study the Riemman hypothesis. In this discipline, I solved a conjecture of Koyama-Kurokawa:

- Norihiko Minami, On the random variable
  $\{1, 2 \ldots, , n\}^r \ni (k_1, k_2, \ldots, k_r) \mapsto gcd(n, k_1, k_2, \ldots, k_r) \in \mathbb{N}$,
  Journal of Number Theory, 133 (2013), no. 8, 2635–2647,

and wrote some other papers, but I had realized essential difficulties, once again.

On the other hand, I had also studied the differentiable 4-manifolds applying the so-called Bauer-Furuta Seiberg-Witten invariants, which value in equivariant stable cohomotopy groups, and wrote some papers with Furuta, Kametani and Matsue.

However, in "§6, Nilpotency rules!"of

- Miko Furuta, Yukio Kametani, Hirofumi Matsue, Norihiko Minami, Homotopy theoretical considerations of the Bauer-Furuta stable homotopy Seiberg-Witten invariants, Geometry & Topology Monographs 10 (2007) 155-166,

I came to the conclusion that, no matter how we work on the Seiberg-Witten equations, we can never solve the Yukio Matsumoto 11/8-conjecture, the most important unsolved problem of the 4-manifolds, by applying the traditional homotopy theoretical technique like the Bauer-Furuta stable homotopy Seiberg-Witten invariants,

Fortunately, we see the light at the end of the tunnel. When the base field k is a subfield of the field of the complex numbers, the Morel-Voevodsky motivic homotopy theory is easily seen to subsume the traditional homotopy theory. So, what I should look after is to investiage global propers of this motivic homotopy theory, which contains truly deep information. Then, in

- Norihiko Minami, From Ohkawa to strong generation via approximable triangulated categories - a variation on the theme of Amnon Neeman's Nagoya lecture series, 16-88, Bousfield classes and Ohkawa's theorem. Springer Proceedings in Mathematics & Statistics, 309. Springer, 2020.,

I formated such a general probllem for general triangulated categories, which I christened as "a hoimework of Tetsusuke Ohkawa."In the case of the motivic stable homotopy category, thisis nothing but the truly difficut problem of classifying the thick ideals. Still, a hint is supplied by the Hopkins-Smoth thick ideal classification theorem for the classical stable homotopy category. Since the Hopkins-Smith thick ideal classification is formuated in terms of some hierarchical structure, we may expect thick ideals of the motivic stable homotopy category are also formulated in terms of some hierarchical structure. Since the motivic stable homotopy theory is an abstract homotopy theory of algebraic geometry, I had realized what I have to do:

Investigate hierarchical structures of algebraic geometry!

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