Research Plan

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1. Geometric Analysis of Polar actions

An isometric action of a Lie group on a Riemannian manifold is called *polar* if there exists a submanifold which meets every orbit orthogonally. Polar actions have a long history and have been studied by many researchers by various methods in Morse theory, Lie theory and so on. In my research, I will study polar actions by the method of geometric analysis, namely by lifting the actions to Hilbert spaces. The purpose of my research is to give a geometric proof of the fact that polar actions on simply connected Riemannian manifolds by connected Lie groups have no exceptional orbits, and to give a classification-free geometric proof of the fact that polar actions on symmetric spaces of higher rank are hyperpolar.

2. Unique existence of minimal orbits in Hermann actions

For a given isometric action of a Lie group on a Riemannian manifold it is a fundamental to determine its minimal orbits. It is known that for the isotropy representation of a compact symmetric space G/K there exists a unique minimal orbit in each strata of the stratification of the orbit types (Hirohashi-Song-Takagi-Tasaki 2000). A similar property also holds for the isotropy action of G/K and more generally for commutative Hermann actions (Ikawa 2011). The purpose of this research is to extend this result to the case of Hermann actions which are not commutative. First I describe the orbits spaces of non-commutative Hermann actions in terms of root systems. If the theorem holds then I will prove it. If it does not hold then I will show a counterexample. This problem is closely related to the problem in the case of affine Kac-Moody symmetric spaces mentioned below and considered to be an important problem.

3. Isotropy representations of affine Kac-Moody symmetric spaces

Affine Kac-Moody symmetric spaces are infinite dimensional symmetric spaces proposed by C.-L. Terng and established by E. Heintze, B. Popescu and W. Freyn based on Kac-Moody theory. Many similar properties between those spaces and finite dimensional Riemannian symmetric spaces are known. In particular their isotropy representations are described by path group actions induced by Hermann actions or σ-actions studied in [5] and [6]. In this research I study the unique existence for minimal orbits in the isotropy representations of affine Kac-Moody symmetric spaces. In the finite dimensional case it is known that there exists a unique minimal orbit in each strata of the stratification of orbit types. I conjecture that the similar property also holds for affine Kac-Moody symmetric spaces. I aim to investigate and prove this conjecture. Moreover I study the symmetries of those minimal orbits, compare them with those in the finite dimensional case and clarify the differences and similarities.

4. Reformulation of soliton theories by affine Kac-Moody groups

According to the research by M. Kashiwara, M. Jimbo, E. Date and T. Miwa, the symmetries of soliton equations are described in terms of affine Kac-Moody algebras. On the other hand, C.-L. Terng and K. Uhlenbeck showed that transformations on solution spaces of some integrable equations can be described by loop group actions. These two researches have been conducted independently and the relation of those is not clear. An *affine Kac-Moody group* is the group corresponding to an affine Kac-Moody algebra and recently its realization is established in the framework of affine Kac-Moody symmetric spaces. An affine Kac-Moody group is realized as a T^2-bundle over a twisted loop group of smooth loops, which has a structure of tame Fréchet manifold (Hamilton 1982) and seems to be useful to study the connection between those two soliton theories. I aim to integrate the above two methods for soliton theories by affine Kac-Moody groups.