Future research plans

We will continue our research about functional equations of the Nekrasov partition function and its variants. The wall-crossing formula is a expression between the coefficients of the partition function and is described by complicated combinatorial coefficients. Therefore, to obtain a functional equation, it is necessary to further evaluate the wall-crossing formula and summarise it into binomial coefficients.

By the wall-crossing formula in a general setting, it will be easier to apply the techniques we have developed to a wide range of areas. Specifically, we are considering an extension to two types of affine A series. One is the partition function of the A_N-type singularity, and the other is the Affine Laumon partition function associated with parabolic vector bundles. The quiver that gives these distribution functions is an extension of the Jordan quiver that gives the most basic Nekrasov partition function, and there is a way to reduce the wall-crossing phenomenon to one for the Jordan quiver. The technical issue is to apply this to equivariant integrals and to study general A-type series. This extends the results for the $A^{(1)}{}_1$-type singularity (paper 5 in the paper list) and the A_1-type Laumon partition function (paper 7 in the paper list).

So far I have not studied the wall-crossing formula on the opposite side, but now using a general framework, I am ready to proceed with the calculations.

I have the research topics from a longer-term perspective.

Generalizing the wall-crossing formula: We want to include flag manifoles other than type A, and framed moduli of vector bundles on 2-dimensional toric stack.

Cohomology theory：We want formulate wall-crossing formulas in terms of the quantum cohomology, elliptic cohomology.

Geometric Representation theory：For asymptotically free Macdonald functions, co-researcher Prof. Shiraishi studied another expression of

A_2 Laumom partition functions. We checked the compatibility of this expression and wall-crossing phenomena. Based on this observation, we further tried to representation theoretic formulation.