Future Research Plan

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Backgrounds to research plans

As already mentioned in the "Summary of Previous Research", it is important to investigate Rarita-Schwinger fields on manifolds with real Killing spinors. Manifolds with real Killing spinors are classified by C. Bär as either Sasaki-Einstein manifolds, 3-Sasakian manifolds, nearly Kähler manifolds or nearly parallel G_2-manifolds. In peer-reviewed papers [1] and [2], Rarita-Schwinger fields on nearly Kähler manifolds and nearly parallel G_2-manifolds have been clarified, respectively. Investigating Rarita-Schwinger fields on the remaining Sasaki-Einstein manifolds and 3-Sasakian manifolds is very interesting, since I expect that Rarita-Schwinger fields have applications to deformation theory on the respective manifolds.

Research plans

(1) Study Rarita-Schwinger fields on Sasaki-Einstein manifolds

Recently, U. Semmelmann, C. Wang, and M.-Y. Wang obtained the result about the linear stability of Sasaki-Einstein manifolds. Linear stability of Einstein metrics is the notion defined as the second variation of the Riemann functional, called the total scalar curvature, in the TT-direction being negative. Now, their result is that Sasaki-Einstein manifolds are linearly unstable if the Betti number satisfies certain conditions. The method is to check linear instability by calculating the difference between the Laplacian for a "good connection" on Sasakian manifolds and the Laplacian for the Levi-Civita connection, and rewriting the harmonic form. I believe that this method can be applied to investigate Rarita-Schwinger fields on Sasaki-Einstein manifolds. I specifically plan to research the following.

There is a special vector field ξ called the Reeb vector field on $(2n+1)$-dimensional Sasaki-Einstein manifolds. The tangent bundle decomposes as $TM = D \oplus \langle \xi \rangle$, where D is the $2n$-dimensional normal bundle of $\langle \xi \rangle$ and is equipped with a transversal Kähler structure. Furthermore, it is known that the spinor bundle $S_{1/2}$ decomposes into a sum of certain irreducible vector bundles by using this transversal Kähler structure and real Killing spinors. Thus the spin-3/2 spinor bundle $S_{3/2} \subset S_{1/2} \otimes TM$ is decomposed into a sum of certain vector bundles.

There exists a "good connection" on Sasaki-Einstein manifolds. I will construct formulas between the twisted Dirac operator and the Rarita-Schwinger operator for this "good connection" and the Levi-Civita connection. Using these tools, I rewrite Rarita-Schwinger fields into tensor products. In fact, I am applying this method to 5-dimensional Sasaki-Einstein manifolds and am working hard on the calculations.

(2) Study Rarita-Schwinger fields on 3-Sasakian manifolds

3-Sasakian manifolds are a special case of Sasaki-Einstein manifolds. I therefore expect to be able to develop the discussion in (1).