Summary of Previous Research

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Backgrounds to the previous research

"Conventional" spin geometry is the area of differential geometry dealing with the Dirac operator and special spinor fields on the spinor bundle $S_{1/2}$. In particular, an element of the kernel of the Dirac operator is called a harmonic spinor. It is known that the existence or non-existence of this harmonic spinor is controlled by the curvature. On the other hand, "new" spin geometry deals with the Rarita-Schwinger operator and Rarita-Schwinger fields on the spin-3/2 spinor bundle $S_{3/2}$, which can be considered analogous to the Dirac operator and harmonic spinors, respectively. Unlike harmonic spinors, the existence or non-existence of Rarita-Schwinger fields depends on the geometric structure and the dimension of the manifold. Thus, the question naturally arises: "On manifolds with what geometric structure can we clarify Rarita-Schwinger fields?". Manifolds with Killing spinors, which are spinor versions of Killing vector fields, are one of the interesting classes that answer this question.

(1) Studied Rarita-Schwinger fields on nearly Kähler manifolds. (peer-reviewed papers [1])

A nearly Kähler manifold is one of manifolds with Killing spinors. I showed that the space of Rarita-Schwinger fields is isomorphic to the space of harmonic 3-forms on 6-dimensional compact nearly Kähler manifolds. This result leads to the existence of a two-dimensional space of Rarita-Schwinger fields on homogeneous and non-homogeneous $S^3 \times S^3$. On the other hand, I found that there is no Rarita-Schwinger field on $S^3 \times S^3$ equipped with the standard metric, which is not a nearly Kähler manifold. This is important because it is the first example where Rarita-Schwinger fields depend not only on the topology but also on the metric.

Furthermore, using a similar method to identify the space of Rarita-Schwinger fields, I also succeeded in identifying the space of infinitesimal deformations of Killing spinors on nearly Kähler manifolds. There is a one-to-one correspondence between the space of Killing spinors and the space of nearly Kähler structures on 6-dimensional spin manifolds. Our result is therefore important as an investigation of the infinitesimal deformation of nearly Kähler structures by A. Moroianu, P.-A. Nagy and U. Semmelmann through Killing spinors.

(2) Studied Rarita-Schwinger fields on nearly parallel G_2-manifolds. (peer-reviewed papers [2])

A nearly parallel G_2-manifold is one of manifolds with Killing spinors. I showed that the space of Rarita-Schwinger fields is isomorphic to a subspace of the eigenspace of the Laplacian on compact nearly parallel G_2-manifolds. Only "negative" results have so far been deduced from the above. In other words, I found that there is no Rarita-Schwinger field on some concrete nearly parallel G_2-manifolds. On the other hand, according to Y. Homma and U. Semmelmann, there are Rarita-Schwinger fields on many torsion-free G_2-manifolds. This means that even on manifolds with the same structure group, the behavior of Rarita-Schwinger fields can be quite different.

Furthermore, using a similar method to identify the space of Rarita-Schwinger fields, I also succeeded in identifying the space of infinitesimal deformations of Killing spinors on nearly parallel G_2-manifolds. There is a one-to-one correspondence between the space of Killing spinors and the space of nearly parallel G_2-structures on 7-dimensional spin manifolds. Our result is therefore important as an investigation of the infinitesimal deformation of nearly parallel G_2-structures by B. Alexandrov and U. Semmelmann through Killing spinors.