Summary of Previous Research Results, Hiroe Oka（岡宏枝）

The dynamical systems theory is a field that attempts to clarify nonlinear phenomena in time-evolving systems in the natural sciences, engineering, social sciences, medicine, etc., using various mathematical tools such as differential equations, topology, ergodic theory, and so on. Many of these methods are based on the construction of mathematical models and their analysis by various mathematical methods, and many results have already been obtained. However, recently, coupled network dynamical systems that appear in life science are large-degree-of-freedom dynamical systems consisting of many nodes, and it is very difficult to understand their dynamics essentially. In addition with the improvement of computer performance, a large amount of numerical data obtained from simulations of differential equations with high-dimensional variables and many parameters, or from experiments and observations of systems for which the model equations are not necessarily clear, have been accumulated. In order to analyze and understand such data, new methods are desired in addition to conventional methods.

I have continued my research on dynamical systems since my Ph. D. dissertation, I applied the normal form theory of dynamical systems to ordinary differential equations of singularly perturbed type and gave a classification for the low codimensional case. These cases include phenomena known from the van der Pol equation and canard phenomena, and show how many parameters are needed for universal perturbations in the case of general vector fields. Since then, I have studied singular perturbations of ordinary differential equations from a geometric point of view, using stable manifolds and other geometrical objects.

For the last decade or so, I have been conducting research on topological computational theory of dynamical systems, which combines topological methods with accuracy-guaranteed numerical calculations to analyze the global structure of dynamical systems with the aid of computers. It is based on the idea of extracting the essential parts of a global structure of the phase-space of complex dynamical systems and expressing them in as concise a form as possible. This method is expected to be effective for network systems with a large number of elements and for the time evolution of image data.

A brief outline of the algorithm used in the theory of topological computation of dynamical systems is as follows:

We take a finite grid partition $\mathcal{G}$ of the phase space of the dynamical system given by the map f. By mapping all grid elements that intersect with the image by f of each grid element, using the outer approximation based on the accuracy-guaranteed computation, we construct a combinatorial multivalued map $\mathcal{F}$ between the grids.

Next, let $\mathcal{G}$ be a set of vertices, and for $G \in \mathcal{G}$, consider an edge from G to H when $H \in \mathcal{F}(G)$. We denote $\mathcal{F}$ as a directed graph. From this directed graph, we can compute topological information including graph algorithms and homology calculations. From this directed graph, the Conley index of the Morse decomposition and each Morse set are obtained, which can be summarized in a simple expression called Conley Morse graph to obtain various mathematically rigorous results on the dynamics of the original dynamical system f.

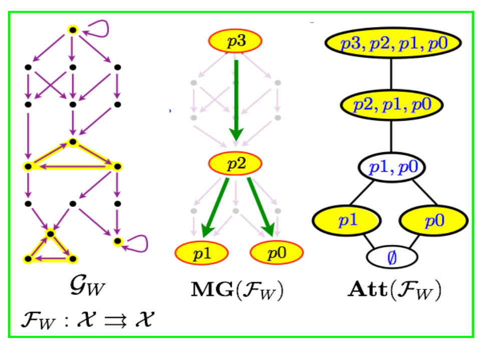

Figure 1: multivalue maps and attractor lattices

By summarizing them into a simple representation called Conley Morse graph, various mathematically rigorous results about the dynamics of the original dynamical system f can be obtained. For a family of parameters of a dynamical system, we call the database of the dynamical system a grid partitioning of the parameter space and phase space, computing a Conley-Morse graph of the corresponding dynamical system for each parameter grid, and the results are aggregated into a searchable form. Such a topological calculation method can be implemented as a software called cmgraph, which assumes the analysis of individual orbits of the target dynamical system, such as fixed points and periodic solutions. It can be calculated almost automatically from the mathematical expression that defines the dynamical system, without assuming the existence of individual orbits such as fixed points and periodic solutions. (Figure 1).

Several results have been obtained based on these ideas.