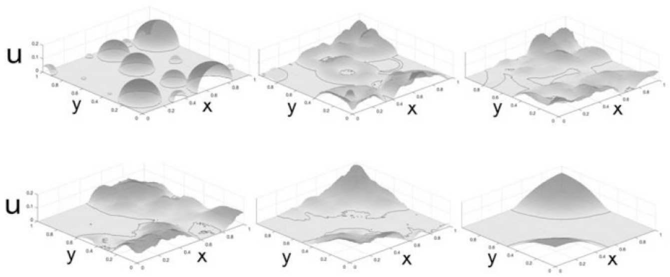
これまでの研究成果のまとめ（小俣正朗）

Research results: (Seiro Omata)

Recent research target is a variational problems especially for treating hyperbolic free boundary problems. Such problems are strongly related to physical models like motion of droplets on a solid obstacle, the motion of bubble on a water surface or the vibration of a string hitting an obstacle. For the sake of simplicity, we just explain the scalar case, where the membrane surrounding the water or the air in the case of a droplet or a bubble can be expressed as the graph of a function.

In general, the membrane forms a positive contact angle with the obstacle, and therefore the Laplacian or other differential operator describing the shape of the membrane is only a measure supported in the boundary of the contact set, usually called ***a free boundary***.




Bubble on water surface (numerical simulation: bubbles are gathering and eventually touch boundary.)

Since theoretical treatment is just on scalar case, however, numerical treatment of the interaction between the membrane and the obstacle can be extended to the vector-valued case.

We here show problems related to wave type equations:

(1) Developing variational treatment (using discrete Morse flow)
(2) Volume constraint problems
(3) bubble motion on the water surface; volume constraint with free boundary
(4) bouncing and rotating elastic shell (roller, tire, etc.)

Our main scheme based on discretizing time. These hyperbolic problems can then be recast as sequences of minimization problems, and this enables us to employ modern techniques from the calculus of variations.

An important aspect is that it develops existence theory constructively, and thus translates directly into numerical approximation schemes.

Although the existence theory that we establish so far is limited to the one-dimensional setting, we note that uniformly bounded approximate solutions can be obtained in arbitrary dimensions.