Summary of Previous Research Results

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The applicant's research topic is the mathematical analysis of the long time behavior of solutions to initial value problems of the nonlinear Schrödinger equation. The applicant has been studying the problem "Under the critical situation where the dispersion due to the linear term and the nonlinearity due to the nonlinear term are balanced, how do the solutions to the corresponding initial value problems behave after a sufficient time has passed? Cubic nonlinear Schrodinger equation in one spatial dimension describing the propagation of optical signals in an optical fiber is in a critical situation in the sense that nonlinear effects remain. The applicant has obtained the following research results from a mathematical point of view.

Lower bound estimates for the lifespan of solutions

① Cubic derivative nonlinear Schrödinger equations in one space dimension.

② Schrödinger equations with subcritical/critical power nonlinearities in 1, 2, and 3 space dimensions.

③ Two-component system of cubic nonlinear Schrödinger equations in one space dimension.

In ①, we consider cubic derivative nonlinear Schrödinger equations in one space dimension, and obtain a detailed lower bound estimate $\liminf_{\varepsilon \to +0} \varepsilon^2 \log T_\varepsilon \geq C_0$. Furthermore, the constant C_0 can be expressed by the imaginary part of a complex-valued function determined by the initial data and nonlinearities. By focusing on the sign change of the imaginary part of the complex-valued function, we were able to summarize previous results on the long-time behavior of the solution from a unified viewpoint. In ②, we extended the results to subcritical /critical power nonlinearities in 1, 2 and 3 space dimensions. Finally, in ③, we attempted an extension to two-component system. By focusing on the conserved quantities of the two-component system, the applicant obtained a detailed lower bound estimates for the two-component system.

Asymptotic behavior of solutions

④ Cubic derivative nonlinear Schrödinger equation in one space dimension.

⑤ Two-component system of cubic nonlinear Schrödinger equations in one space dimension.

In this study, we consider the asymptotic behavior of the solutions to nonlinear Schrödinger equations as t tends to infinity. In ④, derivative nonlinear Schrödinger equation with weakly dissipative structure is considered in terms of L^2-decay rate. As a result, it is found that the derivative nonlinear Schrödinger equation with weakly dissipative structure has an L^2-decay rate exactly intermediate between that of the linear Schrödinger equation and that of the

derivative nonlinear Schrödinger equation with strongly dissipative structure. In ⑤, we consider an initial value problem in which ③ is time-reversed. As a result, we found a new type of large-time behavior in which each of the two components asymptotically approaches a linear solution, but the product of the two components decays to zero at t tends to infinity. The conserved quantities used in ③ are the key to obtaining this result.