Research Plan

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1 Normal Form Theory of Second-Order Equations

As is explained in Research Achievements, it is expected that, by applying the method to prove Borel summability of transformation to the canonical equation near a double turning point, Borel summability of various kinds of transformations will be shown; transformation near a pair of simple turning points (this is proven[7], but a simpler proof will be obtained), transformation near a so-called ghost point, transformation near a triplet of simple turning points, etc. Such transformations are conjectured (claimed) by F. Pham to be resurgent, and Borel summability (if proven) supports the conjecture. This research is on-going. To study such transformations is related to Painlevè equations and important. (In the exact WKB analysis of Painlevè equations, a pair of a double turning point and a simple turning point etc. appear.)

2 Analytic Foundation of the Exact WKB Analysis of Painlevè Equations

The exact WKB analysis of Painlevè equations was vigorously studied by Aoki, Kawai, Takei, et. al. In those days, even Borel summability of WKB solutions of 2nd-order linear ODEs was not established enough, and hence the results are mostly formal. However, later Kamimoto and Koike gave analytic meaning to 0-parameter solutions and 1-parameter solutions. As for 2-parameter solutions, Takei recently proposed a strategy to give an analytic meaning to 2-parameter formal solutions. Also, in the exact WKB analysis of Painlevè equations there naturally appears double turning points in the associated linear equations, and we consider transformation to the canonical equation near a double turning point. As is mentioned above, I gave Borel summability of the transformation near a double turning point (of purely a linear equation). (Note that transformation near a double turning point in the exact WKB analysis of Painlevè equations is more complicated, and this problem is not fully solved yet.) Thus many elements to establish analytic foundation of the exact WKB analysis of Painlevè equations are now prepared, and I aim to achieve it. Specifically, I aim for giving analytic meaning to 2-parameter formal solutions and transformation series to the canonical equation(the first Painlevè equation). The latter one is a natural succeeding research of my result dealing with transformation of a linear equation near a double turning point.

3 Borel Summability of WKB Solutions and Normal Form Theory in Higher-Order Equations

The exact WKB analysis of higher-order equations has recently been used in geometry and mathematical physics, and to solve fundamental problems such as Borel summability of WKB solutions is more desired. I myself had researched toward Borel summability of WKB solutions of higher-order equation, as my main research theme for some years.

However this is an extremely difficult problem which has been unsolved for about 40 years since its importance and difficulty was recognized, and I could not make a significant progress, too. Therefore I will treat this problem only as one my sub-themes for some time, continue analysis of concrete examples by the exact steepest descent method etc. to gain our knowledge, and aim to solve it in the future. Also, beyond this problem, I look ahead normal form theory (especially Borel summability of transformation) of higher-order equations. This is also a natural succeeding research of my results dealing with Stokes phenomena in higher-order equations, bifurcation at a double turning point etc.